

UNIT - IV

Measurements & Engineered Semiconductor Materials

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- **Four-point probe and van der Pauw measurements for carrier density,**
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- **Hot-point probe measurement,**
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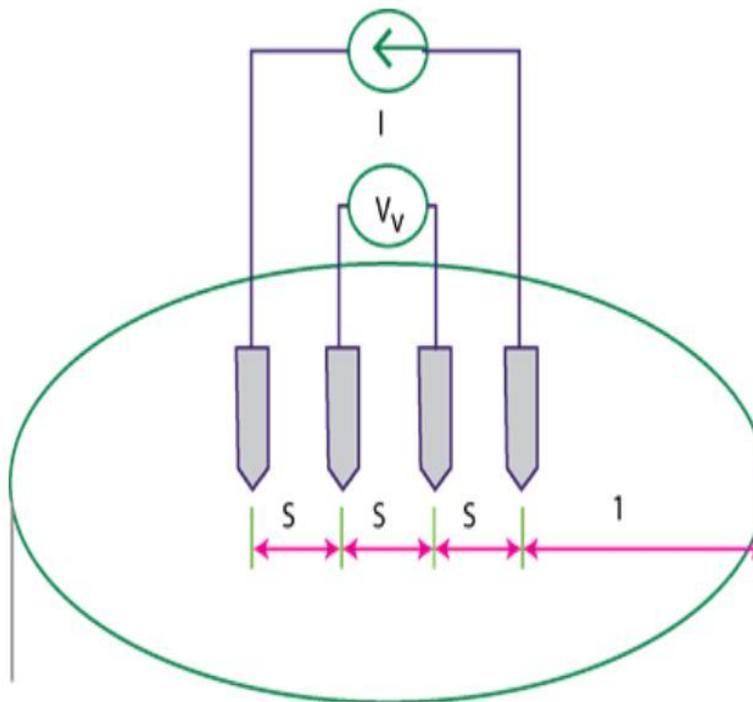
Four-probe method:

Two common techniques used in four-probe method are (i) Four-point collinear probe method and (ii) van der Pauw method.

Four-point collinear probe method:

- This is the most common way of measuring the resistivity of a material, which involves four equally spaced probes as shown in Figure in contact with a materials of unknown resistance.
- No metal contacts are made. Contacts are made by pressing.

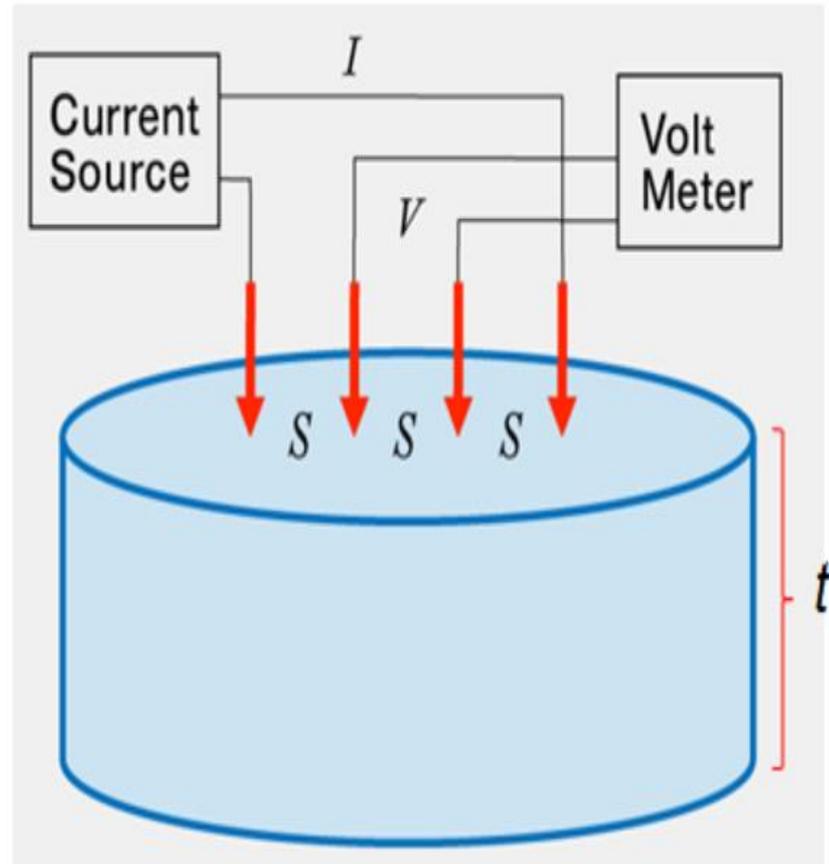
- The four probes are four platinum or gold needle contacts, which is pressed on surface of the sample.
- This method can be used either in bulk or thin film specimen.
- Sample should be regular (shape) in nature.
- Current is applied in the outermost probes and voltage is measured between inner two probes using voltmeter.



For Bulk:

- Consider a bulk material as shown in Figure, where the thickness (t) of the materials is much higher than the space between the probes (s), then the differential resistance due to spherical protrusion of current emanating from the outer probe tips is

$$\Delta R = \rho \left(\frac{dx}{A} \right)$$



Schematic of four-point collinear probe method on bulk material.

Carrying out the integration between the inner probe tips,

$$R = \int_{v_1}^{v_2} \rho \frac{dx}{2\pi x^2} = \int_s^{2s} \rho \frac{dx}{2\pi x^2}$$

$$R = \frac{\rho}{2\pi} \left(-\frac{1}{x} \right) \Big|_s^{2s} = \frac{\rho}{2\pi} \frac{1}{2s}$$

where probe spacing is uniform.

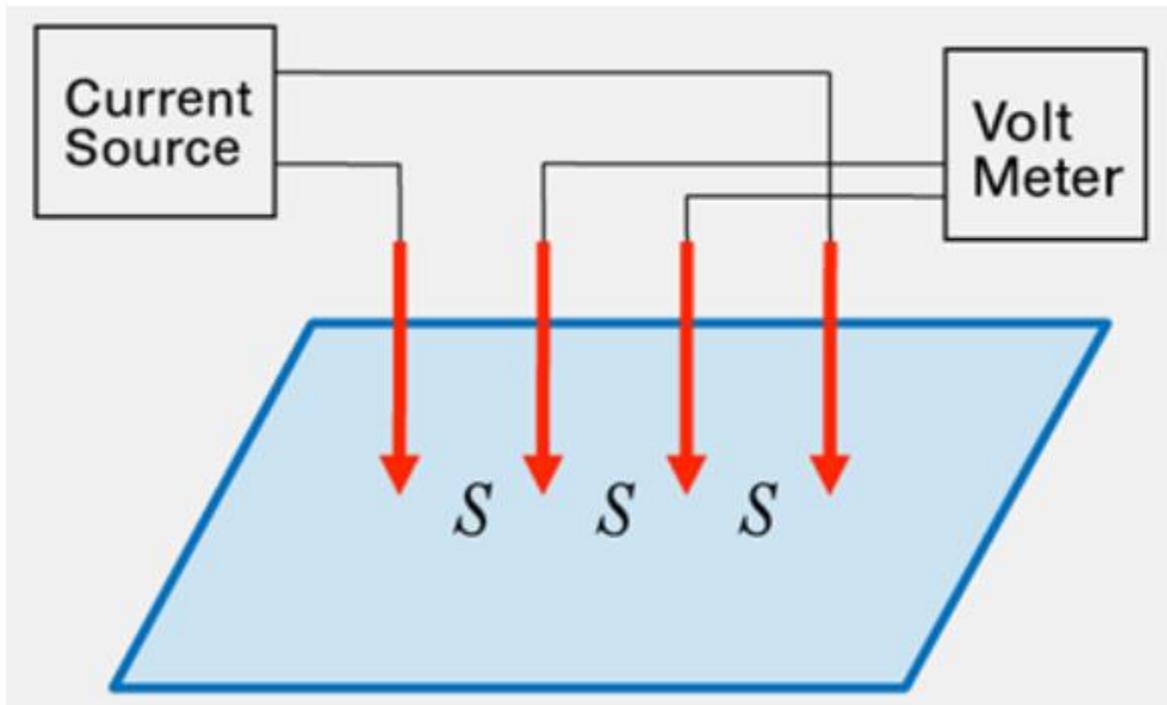
Due to the superposition of current at outer tips, $R = V/(2I)$.

Therefore,

$$\rho = \left(\frac{V}{I} \right) (2\pi s)$$

For Thin sheet:

For a very thin layer as shown in Figure , where the thickness of the sheet $t \ll$ the space between the probes, s , we can get current rings instead of spheres. Therefore, the expression for the area is $A = 2\pi\chi.t$. Therefore, the derivation for resistance turns out to be:



Schematic of four-point collinear probe method on thin sheet.

$$R = \int_{v_1}^{v_2} \rho \frac{dx}{2\pi x t} = \int_s^{2s} \frac{\rho}{2\pi t} \frac{dx}{x}$$

$$R = \frac{\rho}{2\pi t} \ln(x) \Big|_s^{2s} = \frac{\rho}{2\pi t} \ln 2$$

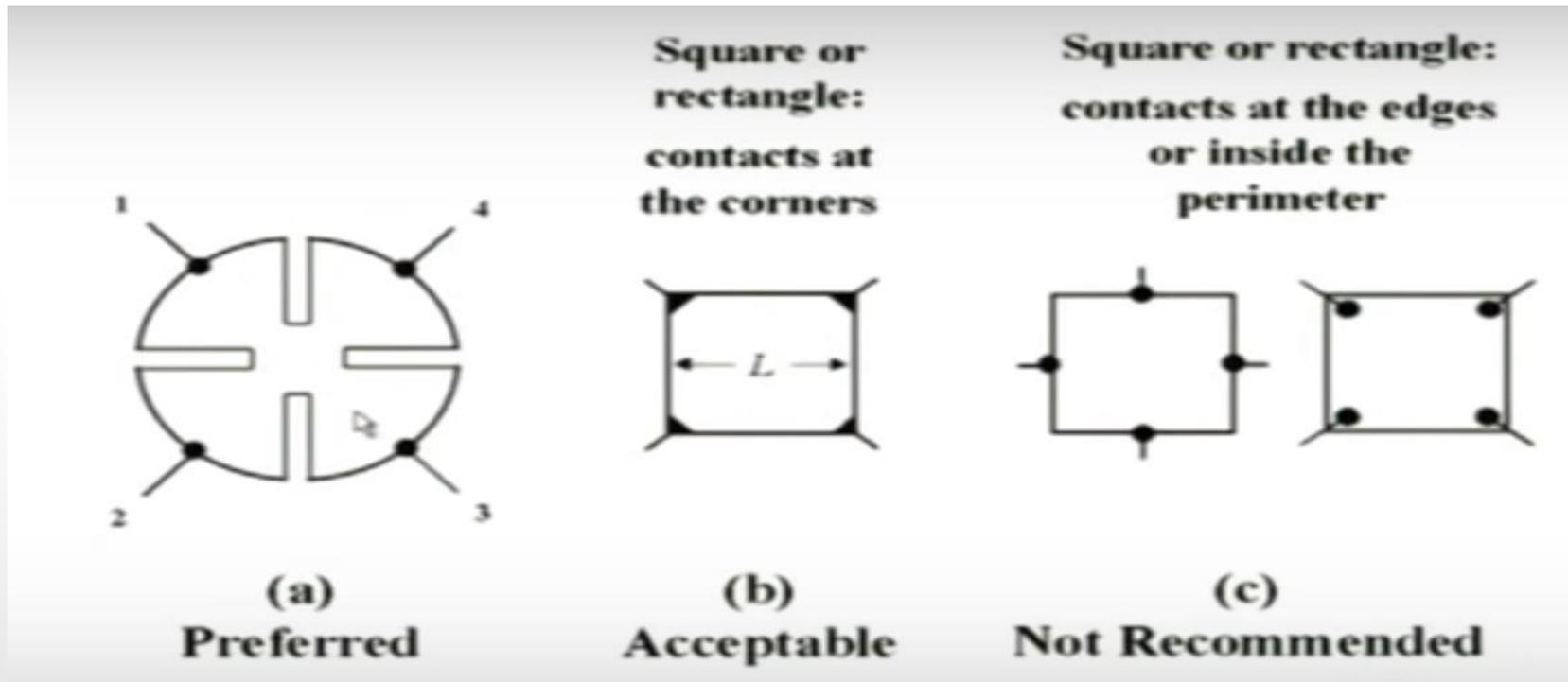
Due to the superposition of current at outer tips, $R = V / (2 I)$.
Therefore, the sheet resistivity for a thin sheet is

$$\rho = \left(\frac{V}{I} \right) \left(\frac{\pi t}{\ln 2} \right)$$

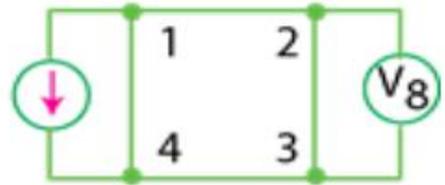
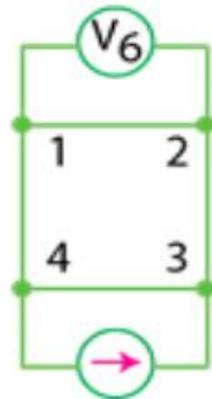
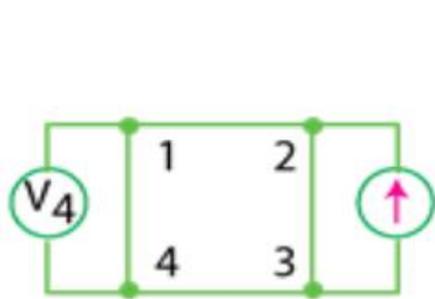
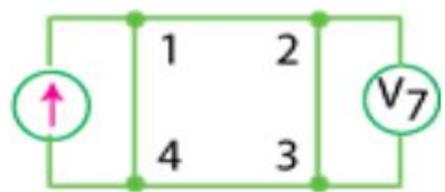
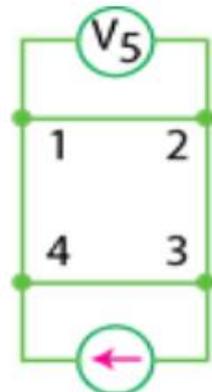
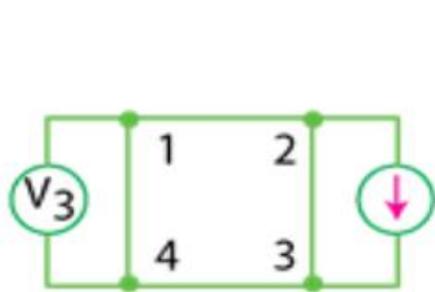
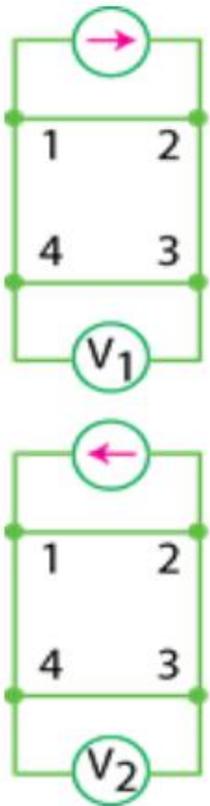
This expression is independent of probe spacing (s). It only depends on thickness (t) of the sample. Here, $(\pi / \ln 2)$ is known as the correction factor.

Van der Pauw method:

- This method involves applying a current and measuring voltage using four small contact on the circumference of a flat, arbitrarily shaped sample of uniform thickness.
- Make Ohmic contacts: contacts by thermal evaporation of conducting material like, gold, silver or aluminium.
- Four very small contacts should be made on the periphery of the sample surface.



- The resistivity can be obtained from a total of eight measurements that are made around the periphery of the sample with the configurations as shown in Figure .



Once all the voltages are taken, two values of resistivity ρ_A and ρ_B can be derived as follows:

$$\rho_A = \left(\frac{\pi}{\ln 2} \right) f_A t_s \left[\frac{(V_1 - V_2 + V_3 - V_4)}{4I} \right]$$

$$\rho_B = \left(\frac{\pi}{\ln 2} \right) f_B t_s \left[\frac{(V_5 - V_6 + V_7 - V_8)}{4I} \right]$$

where ρ_A and ρ_B are volume resistivity in Ohm-cm, t_s is the sample thickness in cm, V_1 to V_8 represent the voltages measured by the voltmeter under eight geometrics respectively, I is the current through the sample in amperes.

- For a perfect symmetry system, $f_A = f_B \approx 1$ and therefore, the average resistivity turns out to be

$$\rho_{avg} = \left(\frac{\rho_A + \rho_B}{2} \right)$$

Hall Mobility Measurements

The basic physical principle underlying the Hall effect is the Lorentz force. When an electron moves along a direction perpendicular to an applied magnetic field, it experiences a force acting normal to both directions and moves in response to this force and the force effected by the internal electric field. The Lorentz force is given by

$$F = q[E + (v \times B)]$$

The Hall effect is illustrated in figure 1 for a bar-shaped sample in which charge is carried by electrons.

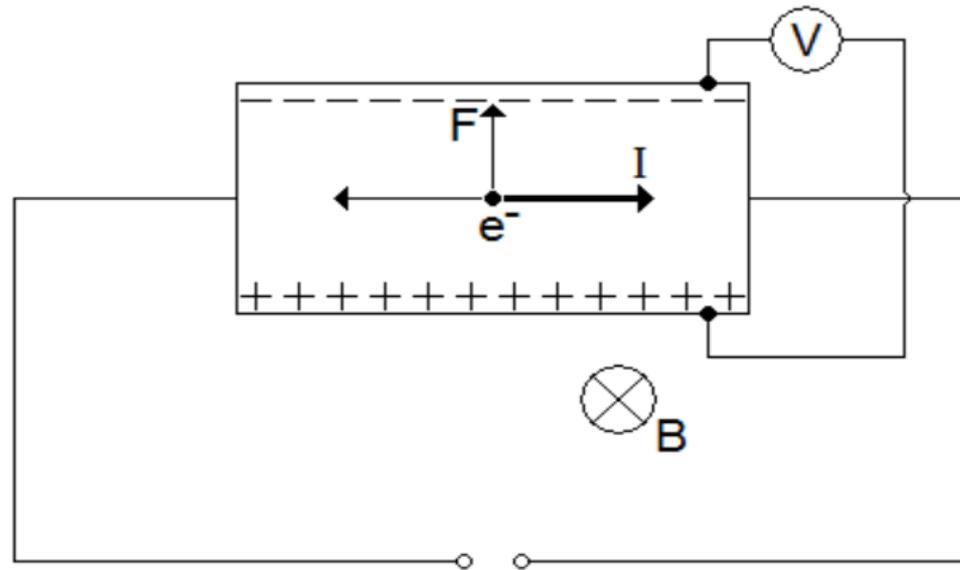


Figure 1: Illustration of the Hall effect in a bar of conducting material.

- A constant current I flows through the bar and the entire bar is subject to a uniform magnetic field \mathbf{B} , which is directed into the screen, perpendicular to the current flow.
- Since the electrons are travelling through a magnetic field, they are subject to an upwards Lorentz force and so drift to the top of the bar whilst maintaining their horizontal motion.
- This leads to a build up of negative charge on one side of the bar and positive charge on the other due to the lack of electrons. This leads to a potential difference between the two sides of the sample, that can be measured as the Hall voltage V_H .
- This transverse voltage is the Hall voltage V_H and its magnitude is equal to IB/qnd , where I is the current, B is the magnetic field, d is the sample thickness, and $q(1.602 \times 10^{-19} \text{ C})$ is the elementary charge. In some cases, it is convenient to use layer or sheet density ($n_s = nd$) instead of bulk density. One then obtains the equation

$$n_s = \frac{IB}{q|V_H|}$$

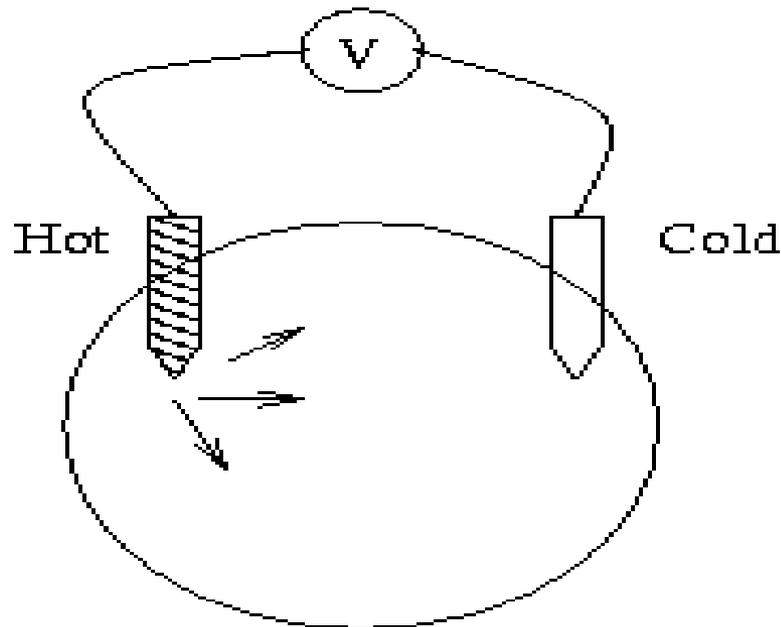
- By measuring the Hall voltage V_H and from the known values of I , B , and q , one can determine the sheet density n_s of charge carriers in semiconductors. If the measurement apparatus is set up as shown, the Hall voltage is negative for n -type semiconductors and positive for p -type semiconductors.
- The sheet resistance R_s of the semiconductor can be conveniently determined by use of the Van der Pauw resistivity measurement technique. Since sheet resistance involves both sheet density and mobility, one can determine the Hall mobility from the equation

$$\mu = \frac{|V_H|}{R_s I B} = \frac{1}{q n_s R_s}$$

If the conducting layer thickness d is known, one can determine the bulk resistivity ($r = R_s d$) and the bulk density ($n = n_s / d$).

Hot point probe measurement

- A conventional Hot-Probe experiment enables a simple and efficient way to distinguish between n-type and p-type semiconductors using a hot probe and a standard voltmeter.
- A hot region of a wafer has a larger number of carriers that will diffuse to cooler regions.



- Thermally excited majority free charged carriers are translated within the semiconductor from the hot probe to the cold probe.
- The mechanism for this motion within the semiconductor is of a diffusion type since the material is uniformly doped due to the constant heating in the hot probe contact.
- These translated majority carriers define the electrical potential sign of the measured current in the multimeter.
- If the carriers are positive (holes), the current will flow in the same direction that the carriers diffuse. Therefore voltmeter will show negative reading.
- If the carriers are negative, the current flow will be in the opposite direction. Therefore, Voltmeter will show positive.
- With the majority carriers diffusing away from the hot probe, the voltage meter will show a positive or negative reading depending on the type of majority carriers in the semiconductor.

Capacitance-Voltage measurements

- Capacitance–voltage measurements is a technique for characterizing semiconductor materials and devices.
- The applied voltage is varied, and the capacitance is measured and plotted as a function of voltage.
- The technique uses a metal–semiconductor junction (Schottky barrier) or a p–n junction to create a depletion region, a region which is empty of conducting electrons and holes, but may contain ionized donors and electrically active defects or traps.
- The depletion region with its ionized charges inside behaves like a capacitor.

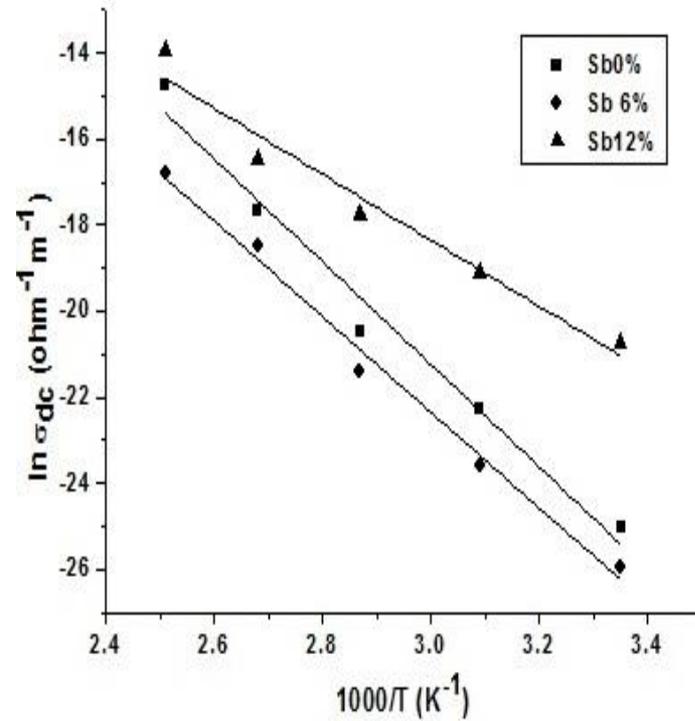
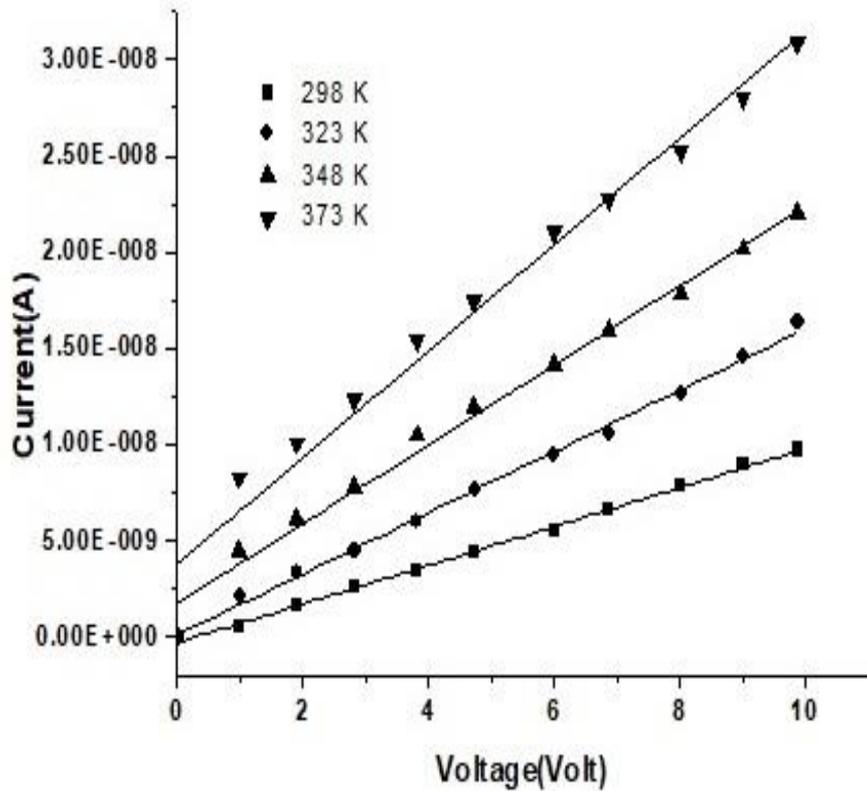
- C–V measurements yield accurate information about doping concentrations of majority carriers as a function of distance (depth) from the junction. By varying the voltage applied to the junction it is possible to vary the depletion width.
- The dependence of the depletion width upon the applied voltage provides information on the semiconductor's internal characteristics, such as its doping profile and electrically active defect densities.
- Measurements may be done at DC, or using both DC and a small-signal AC signal.

Parameter extraction from I-V Characteristics

- The variation of current with voltage represents the I-V characteristics.
- The I-V curves can be recorded on electrometer and the linear fit according to ohm's law ($I=V/R$) and slope ($1/R$) of the curves can be used to evaluate the dc electrical conductivity of the samples using the following relation:

$$\sigma_{dc} = \frac{L}{RA}$$

Where, R denotes resistance, L and A are thickness and cross sectional area of the sample respectively.



The plot of $1000/T$ versus $\ln \sigma_{dc}$ is drawn and the plots are fitted to straight line and slope of the straight line gives activation energy of conduction (ΔE) while the intercept gives pre-exponential factor (σ_o) according to the following relation:

$$\sigma_{dc} = \sigma_o \exp \left[\frac{-\Delta E}{KT} \right]$$

- Pre-exponential factor (σ_0) is helpful in identifying the conduction mechanism taking place in a sample. According to Mott , σ_0 value is two to three orders smaller for conduction in localised states than conduction in extended states.

Hence, the parameters that can be extracted from I-V measurements are:

- 1. Resistance of material (R)**
- 2. Conductivity (σ)**
- 3. Activation energy of conduction (ΔE)**
- 4. Pre-exponential factor (σ_0)**

Deep-level transient spectroscopy (DLTS)

- Deep-level transient spectroscopy (DLTS) is an experimental tool for studying electrically active defects (known as charge carrier traps) in semiconductors.
- DLTS establishes fundamental defect parameters and measures their concentration in the material.
- DLTS investigates defects present in a depletion region of a simple electronic device. The most commonly used are Schottky diodes or p-n junctions.

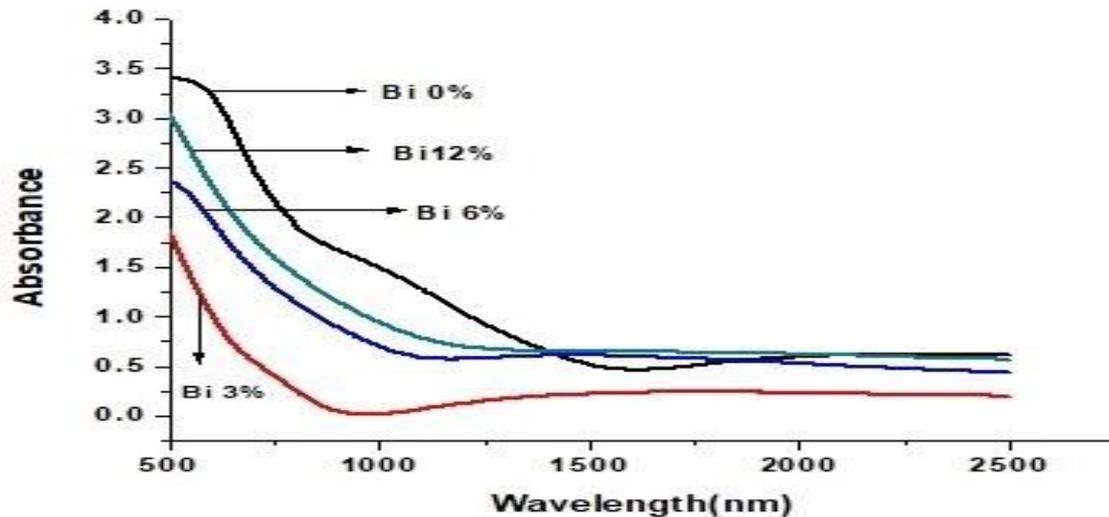
- In the measurement process, the equilibrium state of diode is disturbed by a voltage pulse. This voltage pulse reduces the electric field in the depletion region and allows free carriers from the semiconductor bulk to penetrate this region and recharge the defects causing their non-equilibrium charge state.
- After the pulse, when the voltage returns to its steady-state value, the defects start to emit trapped carriers due to the thermal emission process.
- The technique observes the device depletion region capacitance where the defect charge state recovery causes the capacitance transient.

- The voltage pulse followed by the defect charge state recovery are cycled allowing an application of different signal processing methods for defect recharging process analysis.
- The DLTS technique has a higher sensitivity than almost any other semiconductor diagnostic technique. For example, in silicon it can detect impurities and defects at a concentration of one part in 10^{12} of the material host atoms.

Band gap by UV-Vis spectroscopy, absorption/transmission

- Ultraviolet–visible spectroscopy or ultraviolet–visible spectrophotometry (UV–Vis or UV/Vis) refers to absorption spectroscopy or reflectance spectroscopy in the ultraviolet-visible spectral region. This means it uses light in the visible and adjacent ranges.
- A spectrophotometer is used to record the absorption and transmission spectra of samples in the wavelength range 400-2500 nm at room temperature. The ultraviolet (UV) region scanned is normally from 200 to 400 nm, and the visible portion is from 400 to 800 nm. Above this is infrared region.

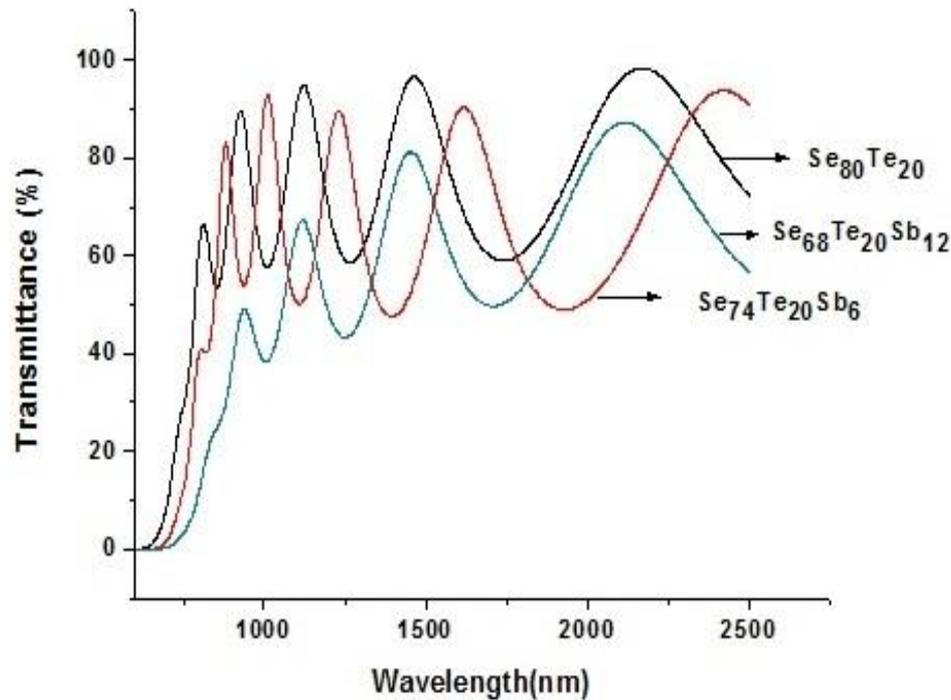
Absorption Spectra: The plot of Absorbance vs wavelength is called Absorption spectra. Absorbance is measure of the capacity of a substance to absorb light of a specified wavelength.



The absorption spectra were utilized to get absorption coefficient (α) given by the following relation:

$$\alpha = 2.302 \left(\frac{\text{Absorbance}}{\text{thickness}} \right)$$

Transmission Spectra: Transmittance of the surface of a material is its effectiveness in transmitting radiant energy. It is the fraction of incident electromagnetic power that is transmitted through a sample.



The Maxima and Minima are obtained corresponding to constructive and destructive interference of light rays interacting with the sample .

Band gap determination

The value of band gap was obtained using Tauc relation, which is The relation between the absorption coefficients (α) and the incident photon energy ($h\nu$) given by

$$(\alpha h\nu) = A (h\nu - E_g)^n$$

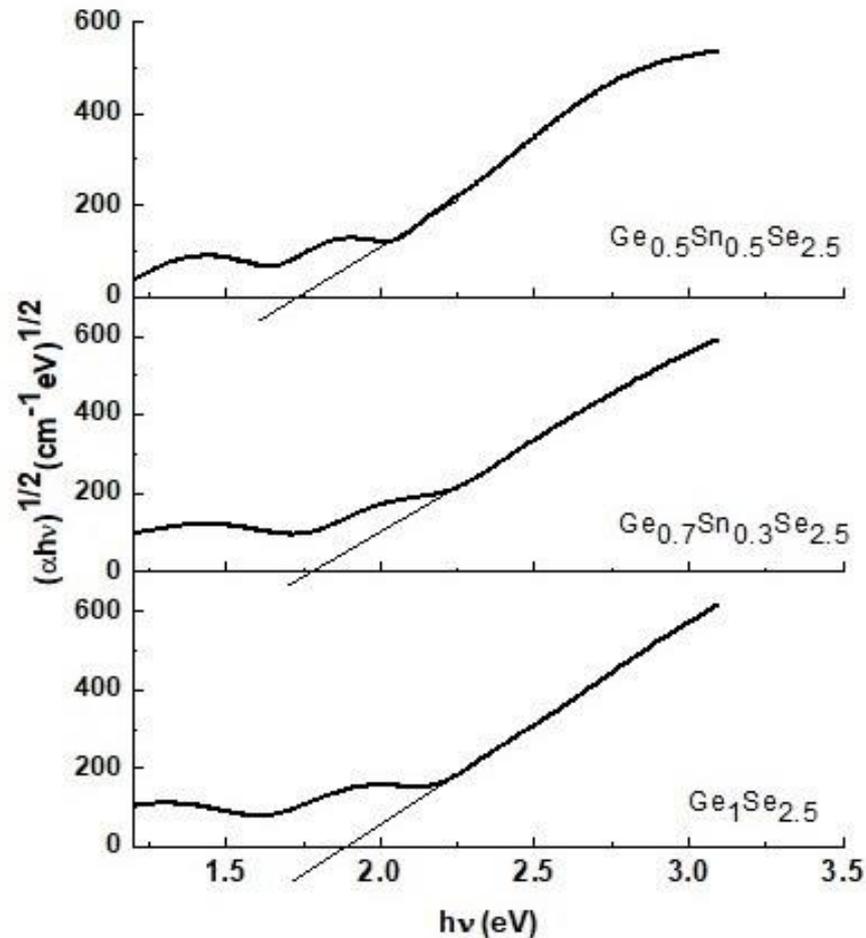
where A is a constant and E_g is the band gap of the material and exponent n depends on the type of transition.

$n = \frac{1}{2}$ for allowed direct transition

$n = 2$ for allowed indirect transition

$n = \frac{3}{2}$ for forbidden transition

Band gap is obtained by extrapolation of the straight line to $(\alpha h\nu)^{1/2} = 0$ axis in the plot of $(\alpha h\nu)^{1/2}$ versus $h\nu$.

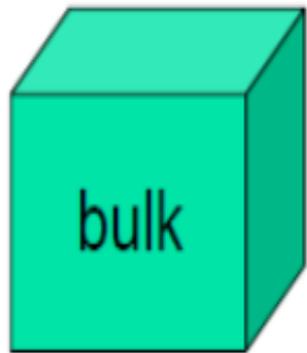


Low-dimensional systems

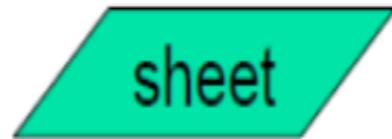
Low-dimensional structures are usually classified according to the number of reduced dimensions they have. More precisely, the dimensionality refers to the number of degrees of freedom in the particle momentum. Accordingly, depending on the dimensionality, the following classification is made:

- Three-dimensional (3D) structure or bulk structure: No quantization of the particle motion occurs, i.e., the particle is free.
- Two-dimensional (2D) structure or quantum well: Quantization of the particle motion occurs in one direction, while the particle is free to move in the other two directions.

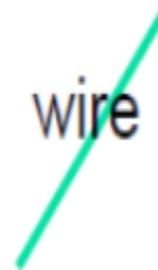
- One-dimensional (1D) structure or quantum wire: Quantization occurs in two directions, leading to free movement along only one direction.
- Zero-dimensional (0D) structure or quantum dot (sometimes called “quantum box”): Quantization occurs in all three directions.



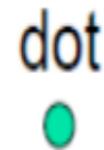
3D



2D



1D



0D

Table 1. Nanostructures and their typical nanoscale dimensions

Nanostructures	Typical nanoscale dimension
Thin films and quantum wells (two-dimensional structures)	1–1000 nm (thickness)
Quantum wires, nanowires, nanorods and nanopillars (one-dimensional structures)	1–100 nm (radius)
Nanotubes	1–100 nm (radius)
Quantum dots, nanodots (zero-dimensional structures)	1–10 nm (radius)

Density of States in 3 D, 2D, 1D and 0D

- The density of states function describes the number of states that are available in a system and is essential for determining the carrier concentrations and energy distributions of carriers within a semiconductor.
- In semiconductors, the free motion of carriers is limited to two, one, and zero spatial dimensions. When applying semiconductor statistics to systems of these dimensions, the density of states in quantum wells (2D), quantum wires (1D), and quantum dots (0D) must be known.

Derivation of Density of States (2D)

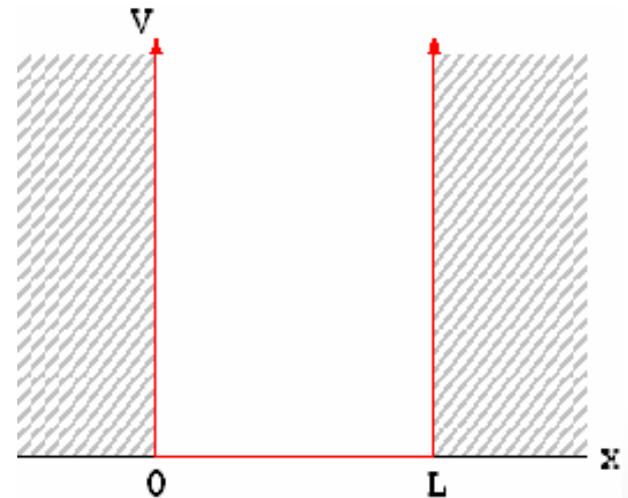
We can model a semiconductor as an infinite quantum well (2D) with sides of length L . Electrons of mass m^* are confined in the well.

If we set the PE in the well to zero, solving the Schrödinger equation yields

$$\left(-\frac{\hbar^2}{2m} \nabla^2 \right) \psi = E \psi$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + k^2 \psi = 0 \quad (\text{Eq. 1})$$

$$\text{where } k = \sqrt{\frac{2mE}{\hbar^2}}$$



Derivation of Density of States (2D)

Using separation of variables, the wave function becomes

$$\psi(x, y) = \psi_x(x) \psi_y(y) \quad (\text{Eq. 2})$$

Substituting Eq. 2 into Eq. 1 and dividing through by $\psi_x \psi_y$ yields

$$\frac{1}{\psi_x} \frac{\partial^2 \psi}{\partial x^2} + \frac{1}{\psi_y} \frac{\partial^2 \psi}{\partial y^2} + k^2 = 0 \quad \text{where } k = \text{constant}$$

This makes the equation valid for all possible x and y terms only if terms including $\psi_x(x)$ and $\psi_y(y)$ are individually equal to a constant.

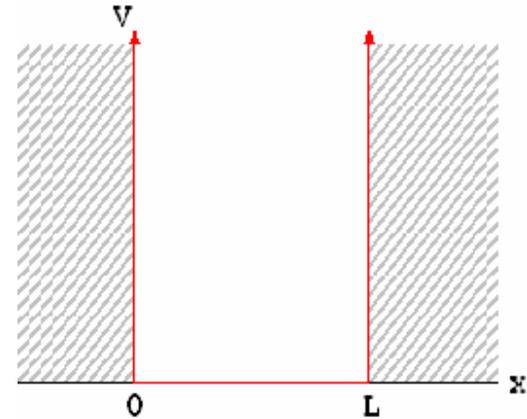
Derivation of Density of States (2D)

Thus,

$$\frac{1}{\psi_x} \frac{\partial^2 \psi}{\partial x^2} = -k^2$$

where $k^2 = k_x^2 + k_y^2$

$$\frac{1}{\psi_y} \frac{\partial^2 \psi}{\partial y^2} = -k^2$$



The solutions to the wave equation where $V(x) = 0$ are sine and cosine functions

$$\psi = A \sin(k_x x) + B \cos(k_x x)$$

Since the wave function equals zero at the infinite barriers of the well, only the sine function is valid. Thus, only the following values are possible for the wave number (k):

$$k_x = \frac{n_x \pi}{L}, \quad k_y = \frac{n_y \pi}{L} \quad \text{for } n = \pm 1, 2, 3, \dots$$

Derivation of Density of States (2D)

Recalling from the density of states 3D derivation...

k-space volume of single state cube in k-space:

$$V_{\text{single-state}} = \left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)\left(\frac{\pi}{c}\right) = \left(\frac{\pi^3}{V}\right)$$

k-space volume of sphere in k-space:

$$V_{\text{sphere}} = \frac{4\pi k^3}{3}$$

$$\text{where } k = \sqrt{\frac{2mE}{\hbar^2}}$$

V is the volume of the crystal.

$V_{\text{single-state}}$ is the smallest unit in k-space and is required to hold a single electron.

Derivation of Density of States (2D)

Recalling from the density of states 3D derivation...

k-space volume of single state cube in k-space: $V_{\text{single-state}} = \left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)\left(\frac{\pi}{c}\right) = \left(\frac{\pi^3}{V}\right) = \left(\frac{\pi^3}{L^3}\right)$

k-space volume of sphere in k-space: $V_{\text{sphere}} = \frac{4\pi k^3}{3}$

Number of filled states in a sphere: $N = \frac{V_{\text{sphere}}}{V_{\text{single-state}}} \times 2 \times \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right)$

$$N = \frac{\frac{4}{3}\pi k^3}{\frac{\pi^3}{L^3}} \times 2 \times \left(\frac{1}{8}\right) = \frac{4\pi k^3 L^3}{3\pi^2}$$

A factor of two is added to account for the two possible electron spins of each solution.

Correction factor for redundancy in counting identical states $\pm n_x, \pm n_y, \pm n_z$

Derivation of Density of States (2D)

For calculating the density of states for a 2D structure (i.e. quantum well), we can use a similar approach, the previous equations change to the following:

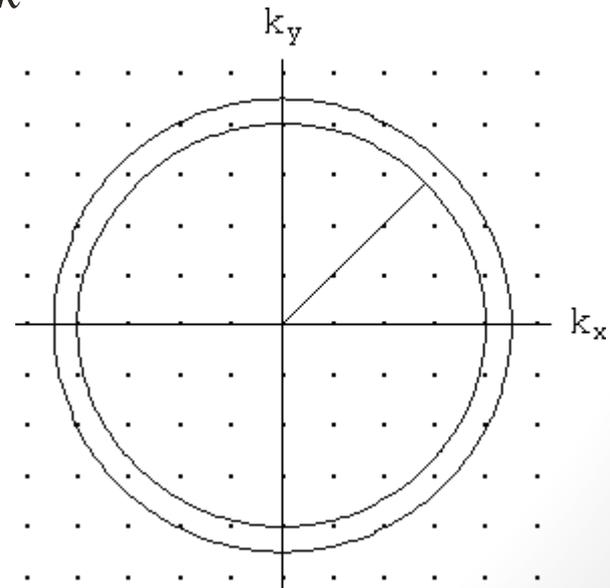
k-space volume of single state cube in k-space: $V_{\text{single-state}} = \left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right) = \left(\frac{\pi^2}{V}\right) = \left(\frac{\pi^2}{L^2}\right)$

k-space volume of sphere in k-space: $V_{\text{circle}} = \pi k^2$

Number of filled states in a sphere:

$$N = \frac{V_{\text{circle}}}{V_{\text{single-state}}} \times 2 \times \left(\frac{1}{2} \times \frac{1}{2}\right)$$

$$N = \frac{\pi k^2}{\frac{\pi^2}{L^2}} \times 2 \times \left(\frac{1}{4}\right) = \frac{k^2 L^2}{2\pi}$$



Derivation of Density of States (2D)

continued..... $N = \frac{k^2 L^2}{2\pi}$, Substituting $k = \sqrt{\frac{2mE}{h^2}}$ yields



$$N = \frac{\left(\sqrt{\frac{2mE}{h^2}} \right)^2 L^2}{2\pi} = \frac{mL^2 E}{h^2 \pi}$$

The density per unit energy is then obtained using the chain rule:

$$\frac{dN}{dE} = \frac{dN}{dk} \frac{dk}{dE} = \frac{L^2 m}{\pi h^2}$$

Derivation of Density of States (2D)

The density of states per unit volume, per unit energy is found by dividing by V (volume of the crystal).

$g(E)_{2D}$ becomes:

$$g(E)_{2D} = \frac{L^2 m}{\pi \hbar^2} = \frac{m}{\pi \hbar^2}$$

As stated initially for the electron mass, $m \rightarrow m^*$.

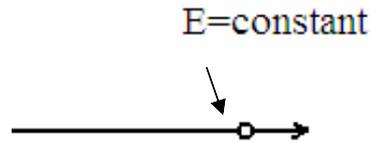
Thus,

$$g(E)_{2D} = \frac{m^*}{\pi \hbar^2}$$

It is significant that the 2D density of states does not depend on energy. Immediately, as the top of the energy-gap is reached, there is a significant number of available states.

Derivation of Density of States (1D)

For calculating the density of states for a 1D structure (i.e. quantum wire), we can use a similar approach. The previous equations change to the following:



k-space volume of single state cube in k-space: $V_{\text{single-state}} = \left(\frac{\pi}{a}\right) = \left(\frac{\pi}{V}\right) = \left(\frac{\pi}{L}\right)$

k-space volume of sphere in k-space: $V_{\text{line}} = k$

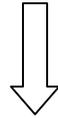
$$N = \frac{V_{\text{line}}}{V_{\text{single-state}}} \times 2 \times \left(\frac{1}{2}\right)$$

Number of filled states in a sphere:

$$N = \frac{k}{\pi} = \frac{kL}{\pi}$$

Derivation of Density of States (1D)

Continued..... $N = \frac{kL}{\pi}$, Substituting $k = \sqrt{\frac{2mE}{h^2}}$ yields



$$N = \frac{\sqrt{\frac{2mE}{h^2}}L}{\pi} = \sqrt{2mE} \frac{L}{h\pi}$$

Rearranging..... $N = (2mE)^{1/2} \frac{L}{h\pi}$

The density per unit energy is then obtained by using the chain rule:

$$\frac{dN}{dE} = \frac{dN}{dk} \frac{dk}{dE} = \frac{\frac{1}{2}(2mE)^{-1/2} \cdot 2mL}{h\pi} = \frac{(2mE)^{-1/2} \cdot mL}{h\pi}$$

Derivation of Density of States (1D)

The density of states per unit volume, per unit energy is found by dividing by V (volume of the crystal).

$g(E)_{1D}$ becomes:

$$g(E)_{1D} = \frac{(2mE)^{-1/2} \cdot mL}{h\pi L} = \frac{(2mE)^{-1/2} \cdot m}{h\pi} = \frac{m}{h\pi\sqrt{2mE}}$$

Simplifying yields...

$$g(E)_{1D} = \frac{m}{h\pi\sqrt{2mE}} \cdot \frac{\sqrt{m}}{\sqrt{m}}$$

$$g(E)_{1D} = \frac{1}{h\pi} \cdot \sqrt{\frac{m}{2E}}$$

Derivation of Density of States (1D)

As stated initially for the electron mass, $m \rightarrow m^*$. Also, because only kinetic energy is considered $E \rightarrow E_c$.

Thus,

$$g(E)_{1D} = \frac{1}{h\pi} \cdot \sqrt{\frac{m^*}{2(E - E_c)}}$$

Derivation of Density of States (0D)

When considering the density of states for a 0D structure (i.e. quantum dot), no free motion is possible. Because there is no k-space to be filled with electrons and all available states exist only at discrete energies, we describe the density of states for 0D with the delta function.

Thus,

$$g(E)_{0D} = 2\delta(E - E_c)$$

Additional Comments

The density of states has a functional dependence on energy.

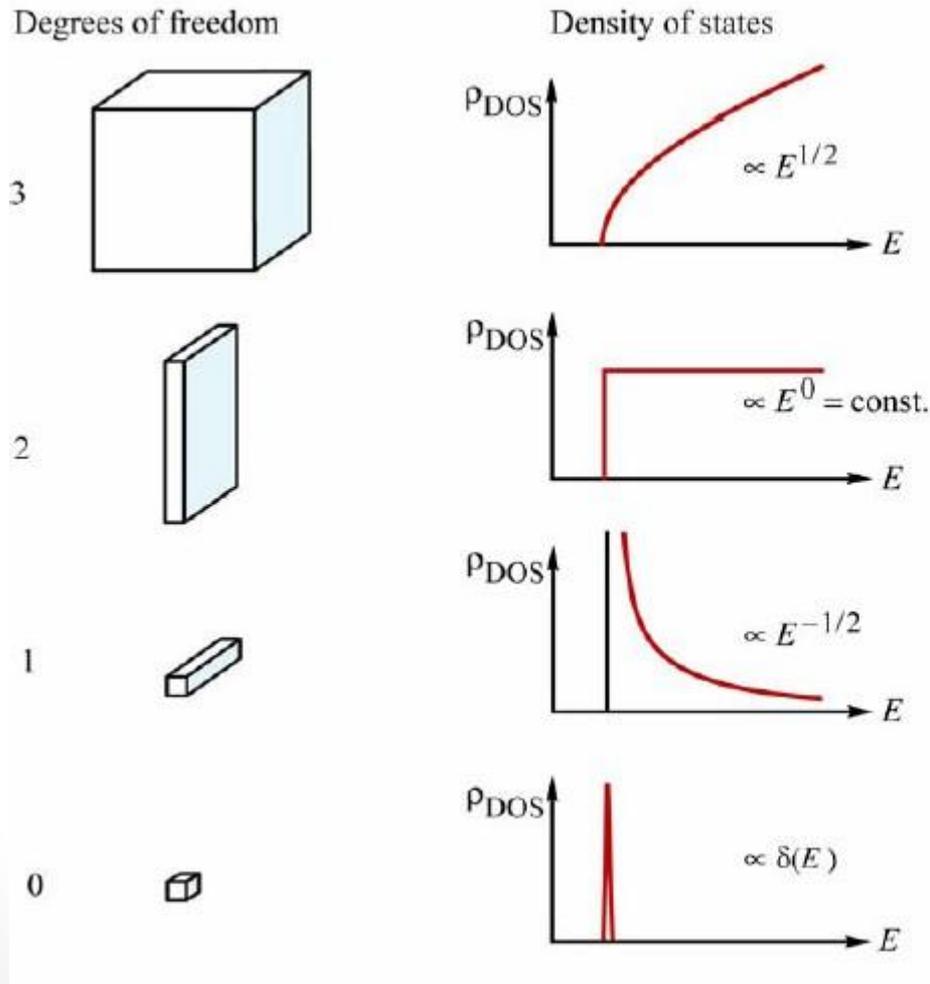


Fig. 12.7. Electronic density of states of semiconductors with 3, 2, 1, and 0 degrees of freedom for electron propagation. Systems with 2, 1, and 0 degrees of freedom are referred to as quantum wells, quantum wires, and quantum boxes, respectively.

Additional Comments

Degrees of freedom	Dispersion (kinetic energy)	Density of states	Effective density of states
3 (bulk)	$E = \frac{\hbar^2}{2m^*}(k_x^2 + k_y^2 + k_z^2)$	$\rho_{\text{DOS}}^{3\text{D}} = \frac{1}{2\pi^2} \left(\frac{2m^*}{\hbar^2} \right)^{\frac{3}{2}} \sqrt{E - E_C}$	$N_c^{3\text{D}} = \frac{1}{\sqrt{2}} \left(\frac{m^* kT}{\pi \hbar^2} \right)^{\frac{3}{2}}$
2 (slab)	$E = \frac{\hbar^2}{2m^*}(k_x^2 + k_y^2)$	$\rho_{\text{DOS}}^{2\text{D}} = \frac{m^*}{\pi \hbar^2} \sigma(E - E_C)$	$N_c^{2\text{D}} = \frac{m^*}{\pi \hbar^2} kT$
1 (wire)	$E = \frac{\hbar^2}{2m^*}(k_x^2)$	$\rho_{\text{DOS}}^{1\text{D}} = \frac{m^*}{\pi \hbar} \sqrt{\frac{m^*}{2(E - E_C)}}$	$N_c^{1\text{D}} = \sqrt{\frac{m^* kT}{2\pi \hbar^2}}$
0 (box)	–	$\rho_{\text{DOS}}^{0\text{D}} = 2\delta(E - E_C)$	$N_c^{0\text{D}} = 2$

Table 12.1 Density of states for semiconductor with 3, 2, 1, and 0 degrees of freedom for propagation of electrons. The dispersion relations are assumed to be parabolic. The formulas can be applied to anisotropic semiconductors if the effective mass m^* is replaced by the density-of-states effective mass m_{DOS}^* . If the semiconductor has a number of M_c equivalent minima, the corresponding density of states must be multiplied by M_c . The bottom of the band is denoted as E_C and $\sigma(E)$ is the step-function.

Practical Applications

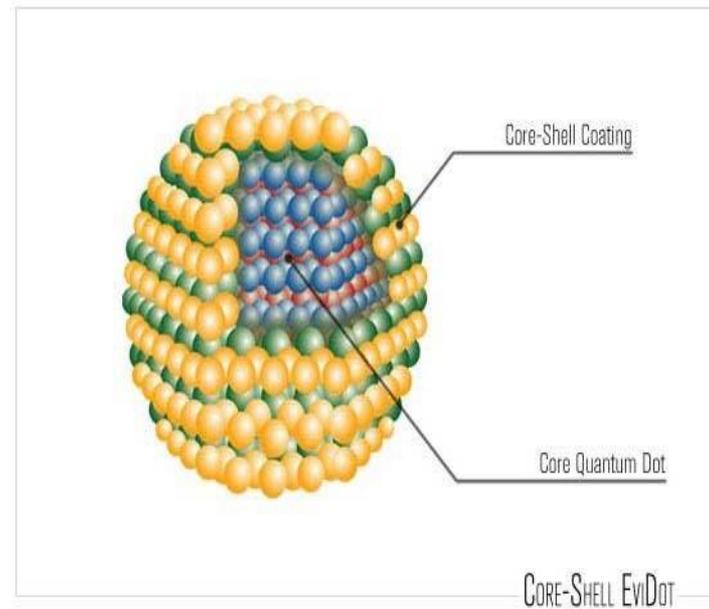
Quantum Wells (2D) - a potential well that confines particles in one dimension, forcing them to occupy a planar region

Quantum Wire (1D) - an electrically conducting wire, in which quantum transport effects are important

Quantum Dots (0D) - a semiconductor crystal that confines electrons, holes, or electron-pairs to zero dimensions.

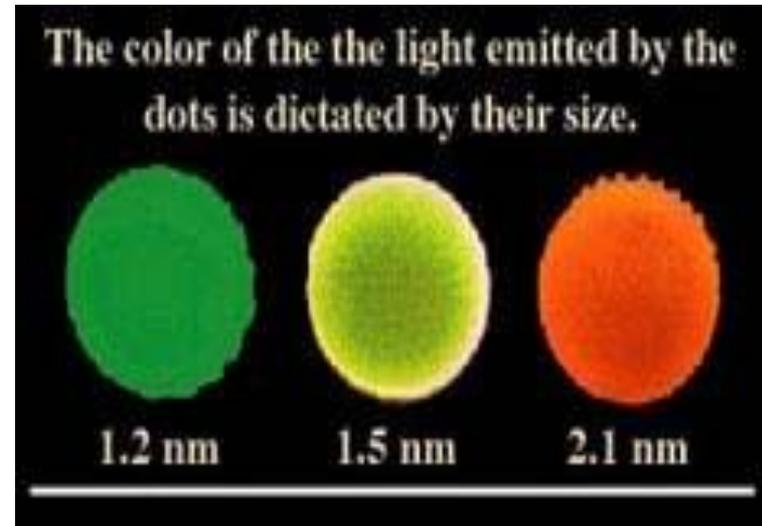
Quantum Dots

- Small devices that contain a tiny droplet of free electrons.
- Dimensions between nanometers to a few microns.
- Contains single electron to a collection of several thousands
- Size, shape, and number of electrons can be precisely controlled



Quantum Dots

- Exciton: bound electron-hole pair (EHP)
- Attractive potential between electron and hole
- Excitons generated inside the dot
- Excitons confined to the dot
 - Degree of confinement determined by dot size
 - Discrete energies



Fabrication Methods

- Goal: to engineer potential energy barriers to confine electrons in 3 dimensions
- 3 primary methods
 - Lithography
 - Colloidal chemistry
 - Epitaxy

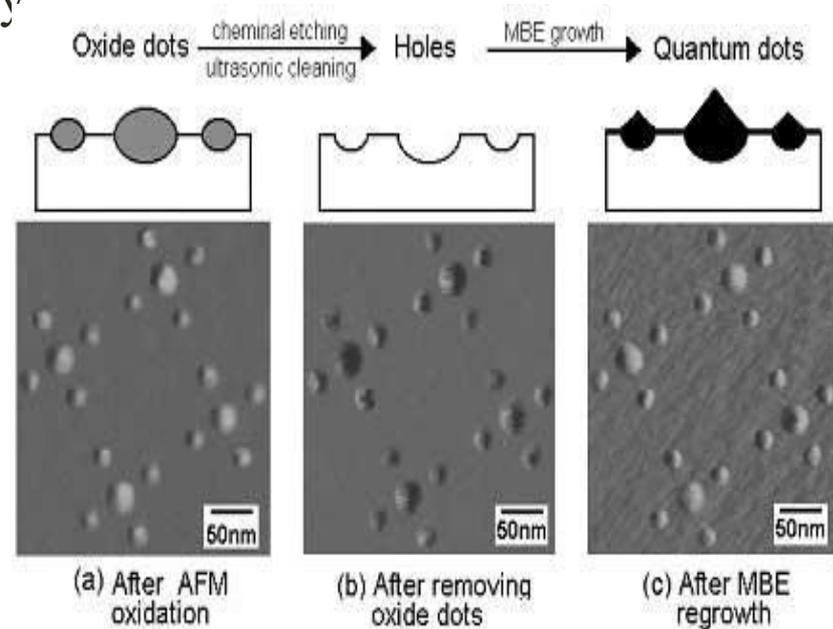


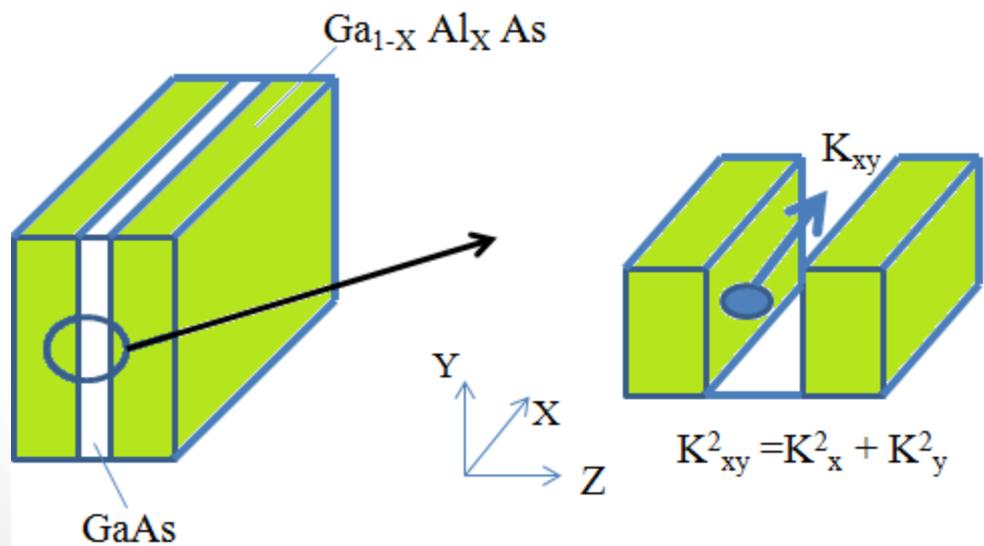
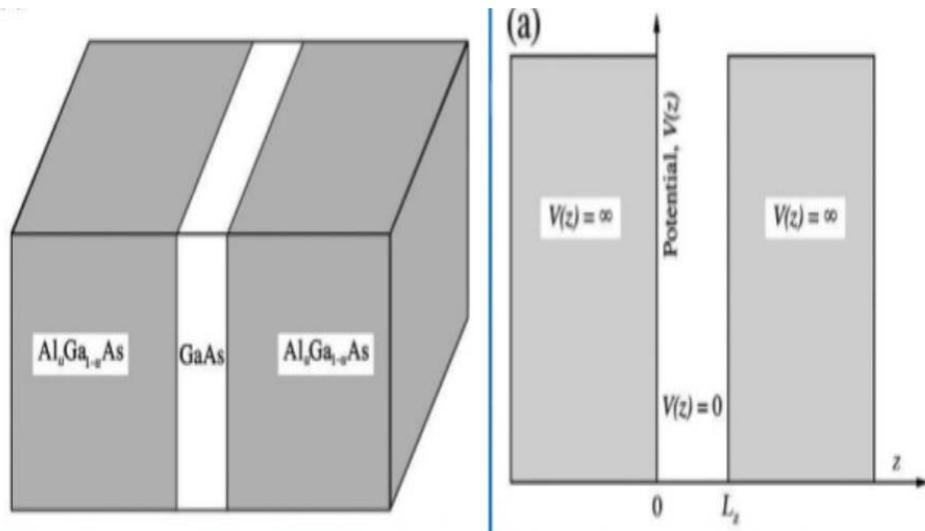
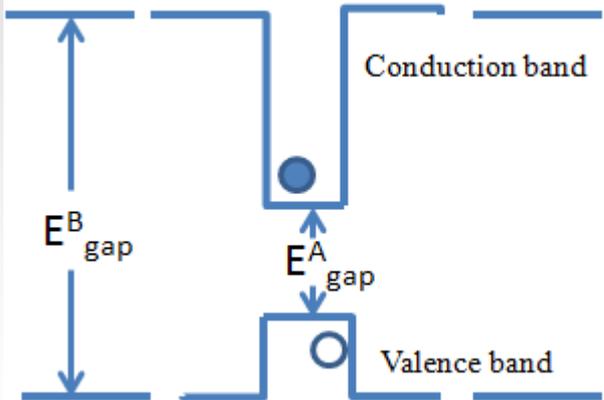
FIG. 1 Quantum dot fabrication processes and array of fabricated quantum dots

Design and fabrication

Quantum well:

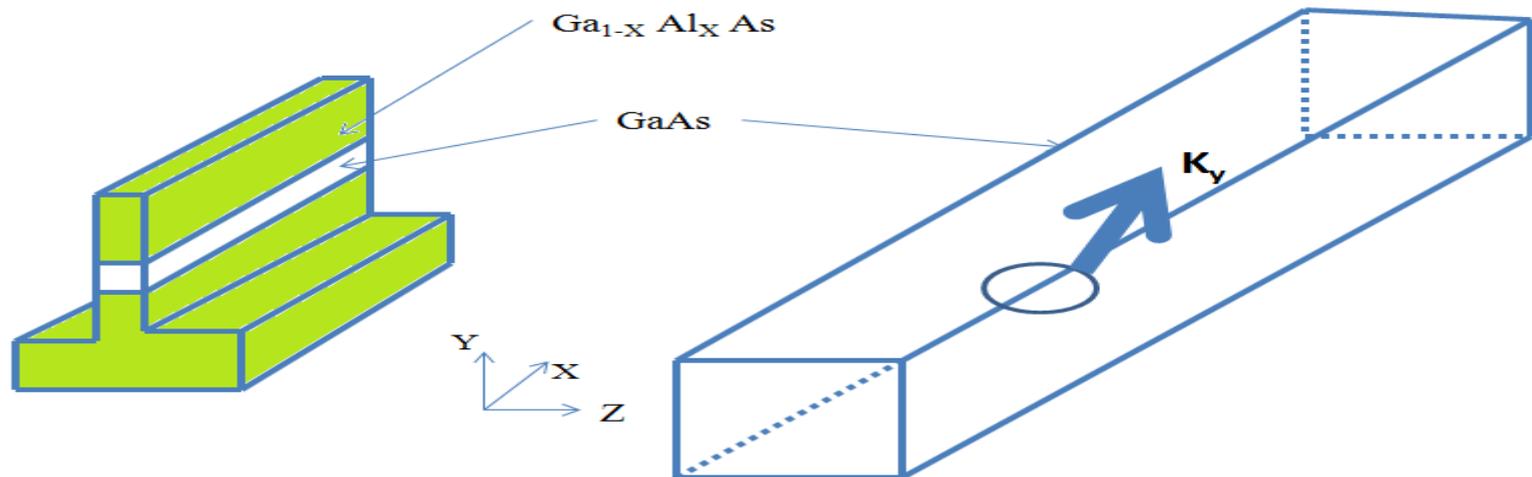
Design: Quantum wells are real-world implementation of the “particle in the box” problem; they act as potential wells for charge carriers and are typically experimentally realized by epitaxial growth of a sequence of ultrathin layers consisting of semiconducting materials of varying composition.

Fabrication: Two dissimilar semiconductors with different band gaps can be joined to form a heterojunction. The discontinuity in either the conduction or the valence band can be used to form a potential well. If a thin layer of a narrower-band gap material 'A' say, is sandwiched between two layers of a wider-band gap material 'B', then they form a double heterojunction. If layer 'A' is sufficiently thin for quantum properties to be exhibited, then such a band alignment is called a single quantum well.



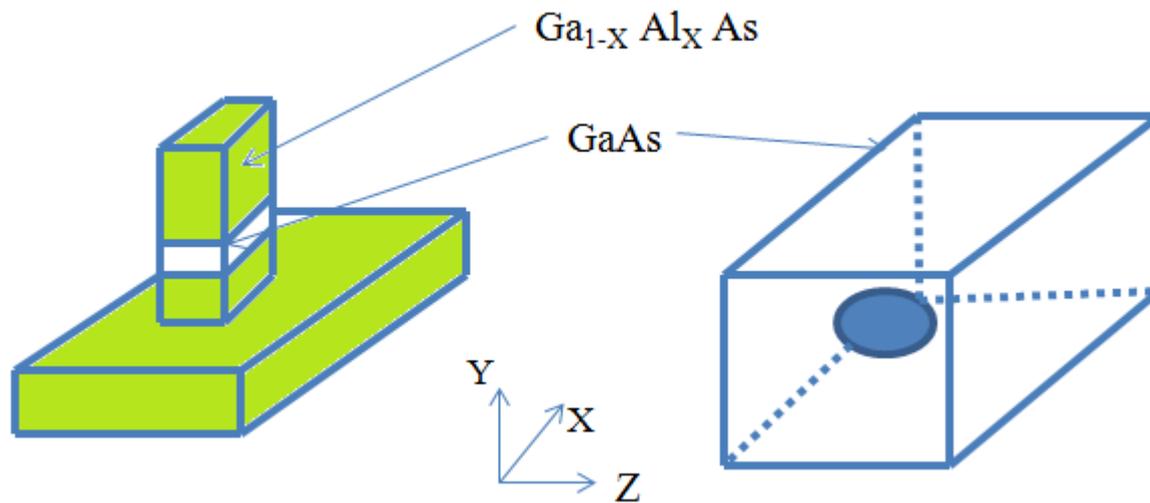
Quantum wire

- A standard quantum well layer can be patterned with photolithography or perhaps electron-beam lithography, and etched to leave a free standing strip of quantum well material.
- The following Fig. shows an expanded view of a single quantum wire, where clearly the electron (or hole) is free to move in only one direction, in this case along the y-axis.



Quantum dot:

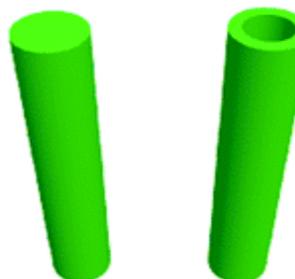
Quantum dots can again be formed by further lithography and etching, e.g. if a quantum well sample is etched to leave pillars rather than wires, then a charge carrier can become confined in all three dimensions, as shown in Fig.



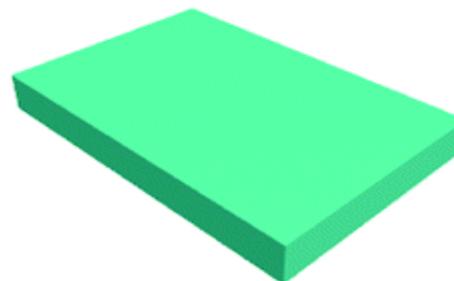
a 0D NPs



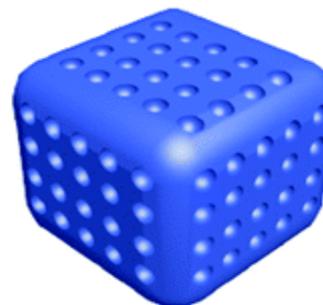
b 1D NWs/NTs



c 2D thin film



d 3D porous



Characterization techniques

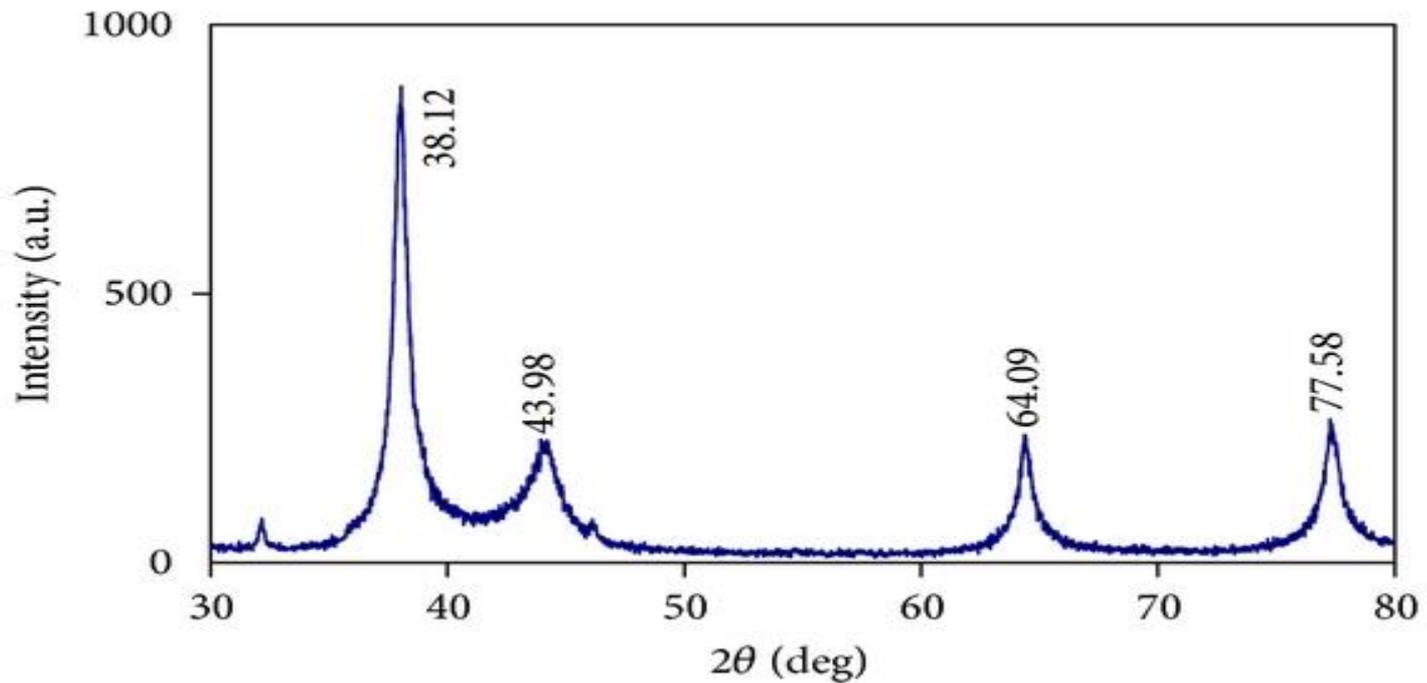
X-ray diffraction

- When an X-ray beam hits an atom, the electrons around the atom start to oscillate with the same frequency as the incoming beam. In almost all directions destructive interference is observed, that is, the combining waves are out of phase and there is no resultant energy leaving the solid sample.
- However, the atoms in a crystal are arranged in a regular pattern, and in a very few directions constructive interference is obtained. The waves will be in phase and there will be well defined X-ray beams leaving the sample at various directions.
- Hence, a diffracted beam may be described as a beam composed of a large number of scattered rays mutually reinforcing one another.

- Bragg obtained the geometrical relationship between wavelength (λ), atomic spacing (d) and angle (θ) is given by as:

$$2d \sin\theta = n \lambda$$

- If the wavelength is known (depends on the type of X-ray tube used) and the angle can be measured (with diffractometer) then the interplanar distance can be calculated from the Bragg's equation.
- The finite size of the crystallites has a marked effect on the X-ray diffraction line widths. With reduction in the particle size, observed X-ray diffraction peaks broaden significantly. This is due to the finite size of the nanoparticles.



The particle size of these nanoparticle using Debye Scherrer formula. The Scherrer formula is given by:

$$D = 0.9 \lambda / \beta \cos \theta$$

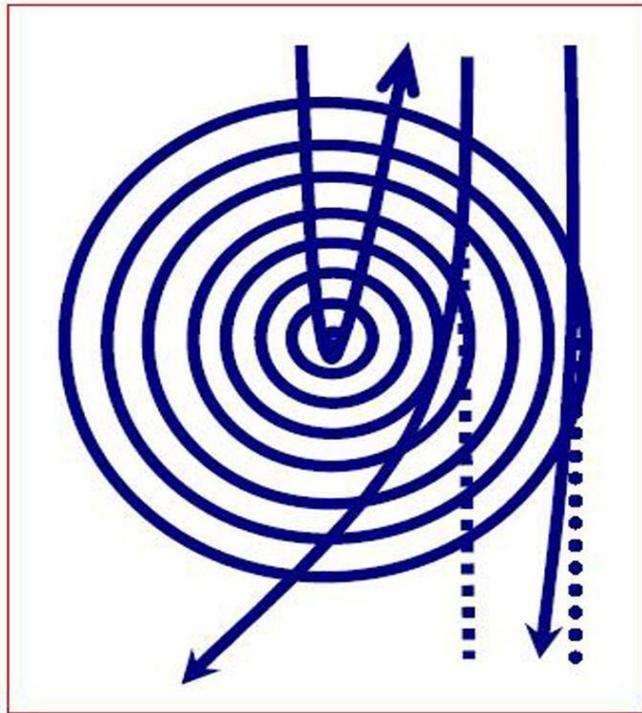
where D is the average particle size perpendicular to the reflecting planes, λ is the X-ray wavelength, β is the full width at half maximum (FWHM), and θ is the diffraction angle.

Transmission Electron Microscopy (TEM)

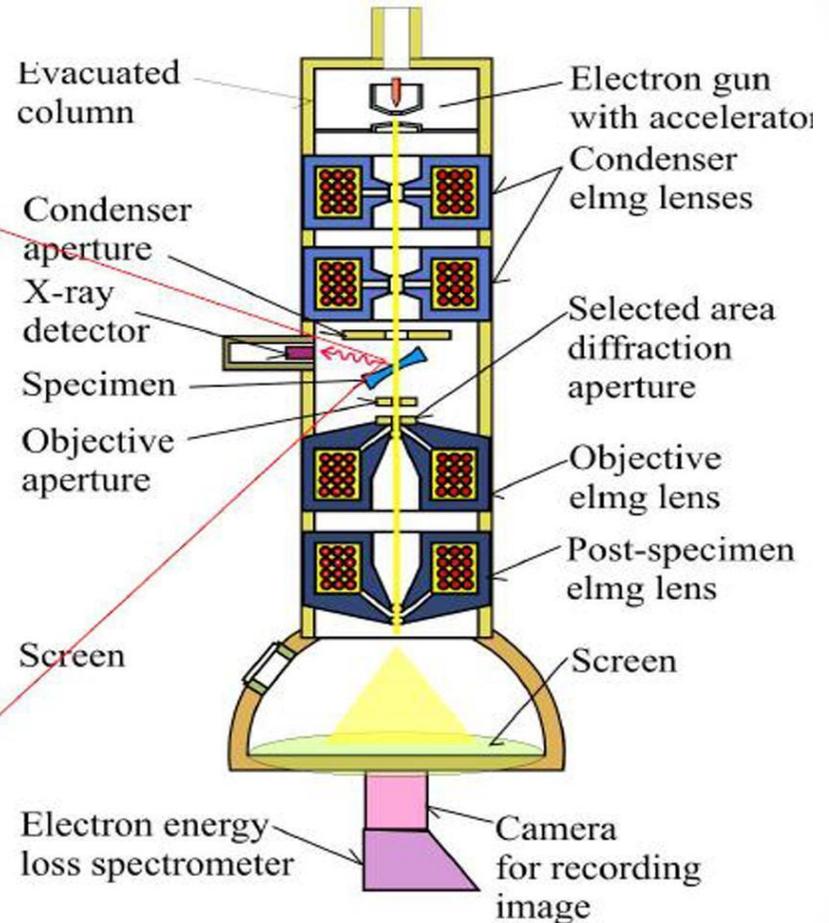
- Transmission Electron Microscopy (TEM) is a versatile technique to study the particle size, shape and size distribution of particles from solutions and dry origins.
- TEM is also useful in the identification of crystal structure of crystalline particles by electron diffraction.
- In Transmission Electron Microscope, a beam of electrons is transmitted through a specimen to form an image. The specimen is most often an ultrathin section less than 100 nm thick or a suspension on a grid.
- An image is formed from the interaction of the electrons with the sample as the beam is transmitted through the specimen. The image is then magnified and focused onto an imaging device, such as a fluorescent screen, a layer of photographic film.

Transmission electron microscope

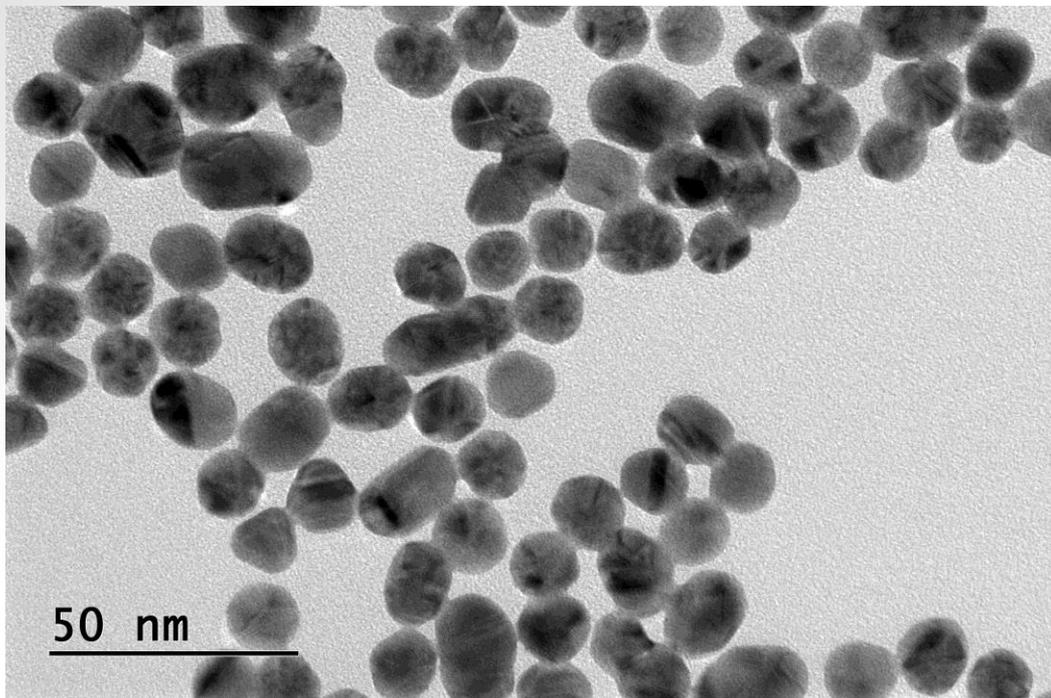
Interaction of electron with sample



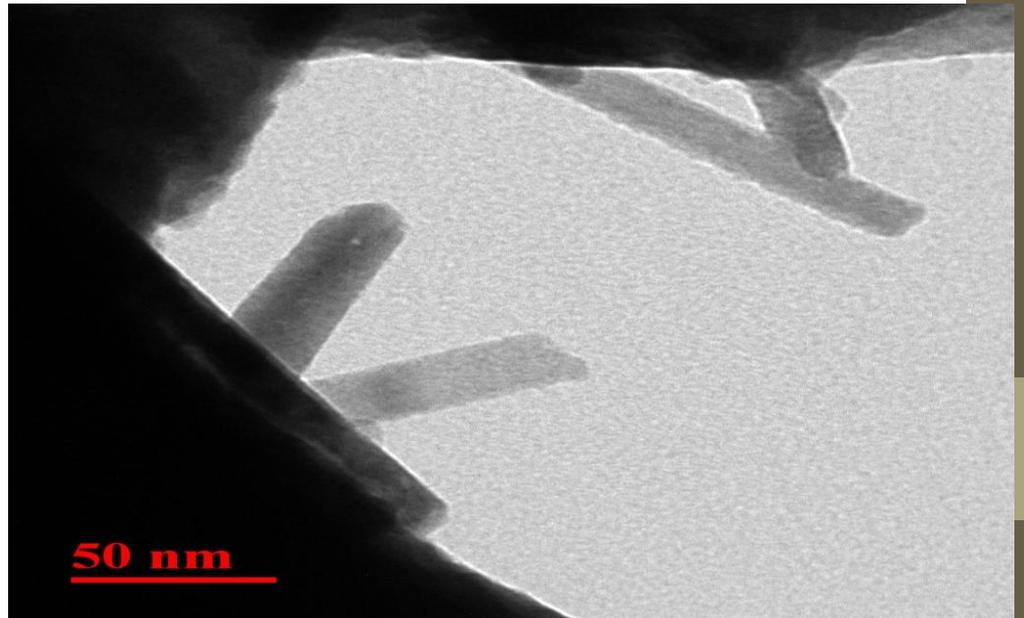
Electron scattering



- The electron beam is confined by the two condenser lenses which also control the brightness of the beam. This electron beam passes the condenser aperture and “hits” the sample surface.
- The electrons that are elastically scattered consist of the transmitted beams, which pass through the objective lens. The objective lens forms the image display and the following apertures, the objective and selected area aperture are used to choose of the elastically scattered electrons that will form the image of the microscope.
- Finally, the beam goes to the magnifying system consisting of three lenses, the first and second intermediate lenses which control the magnification of the image and the third one is projector lens. The formed image is shown either on a fluorescent screen or in monitor or both and is printed on a photographic film.

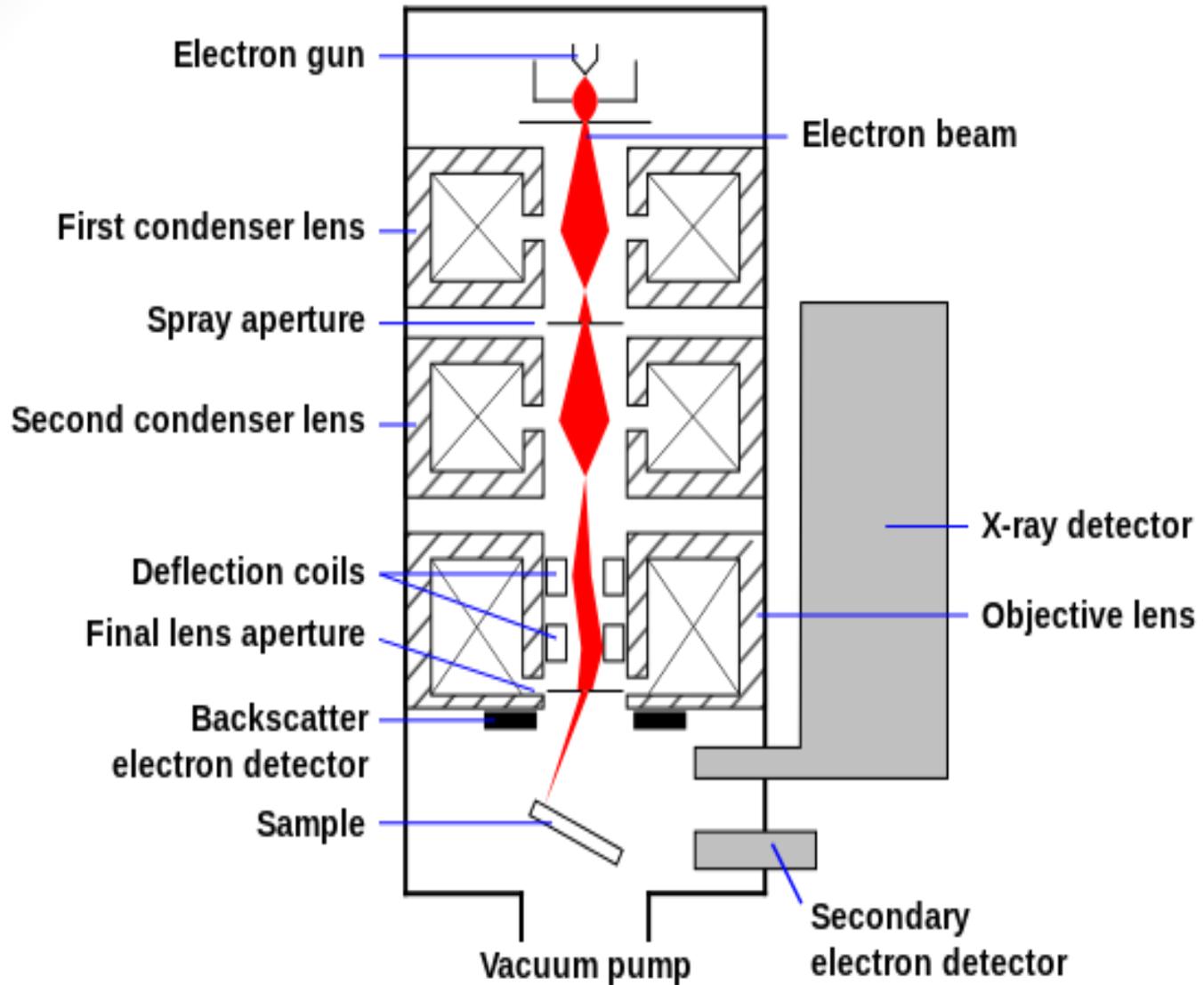


TEM Images



Scanning electron microscope (SEM)

- A scanning electron microscope (SEM) is a type of electron microscope that produces images of a sample by scanning the surface with a focused beam of electrons.
- The electrons interact with atoms in the sample, producing various signals that contain information about the surface topography and composition of the sample.
- The electron beam is scanned in a raster scan pattern, and the position of the beam is combined with the detected signal to produce an image.
- The most common SEM mode is the detection of secondary electrons emitted by atoms excited by the electron beam. The number of secondary electrons that can be detected depends, among other things, on specimen topography.
- By scanning the sample and collecting the secondary electrons that are emitted using a special detector, an image displaying the topography of the surface is created.



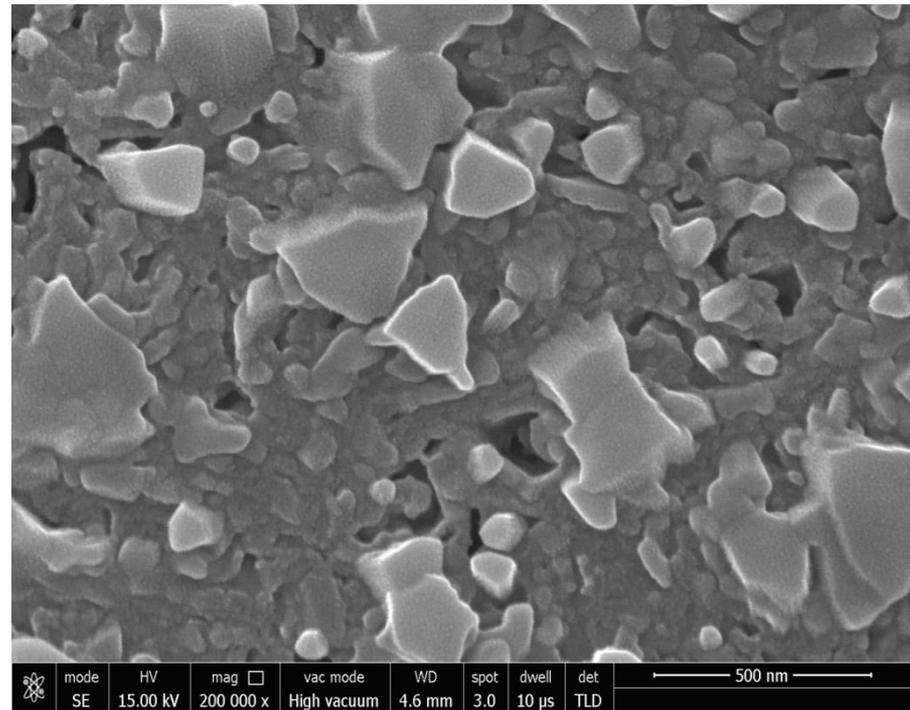
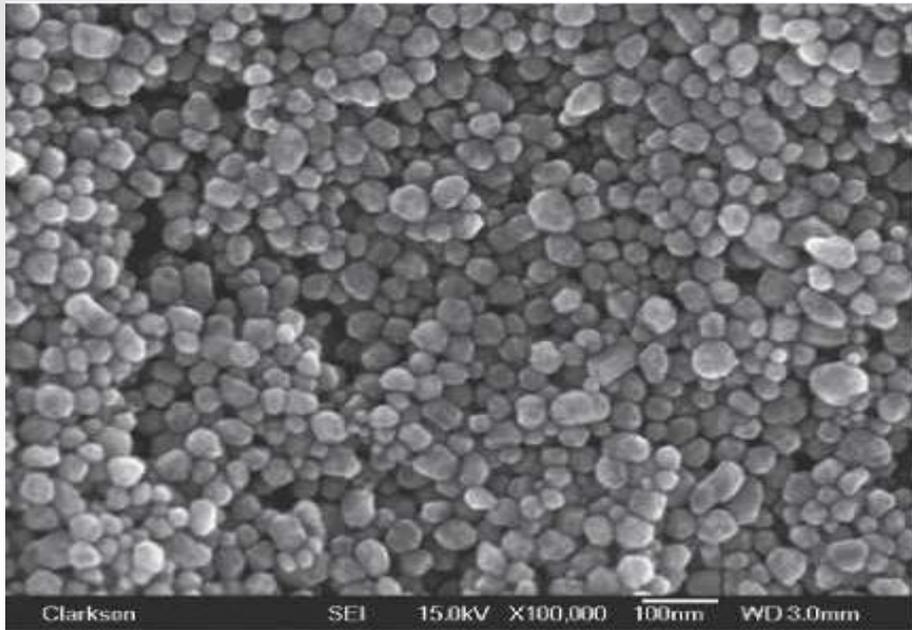
Schematic of SEM

Scanning process and image formation

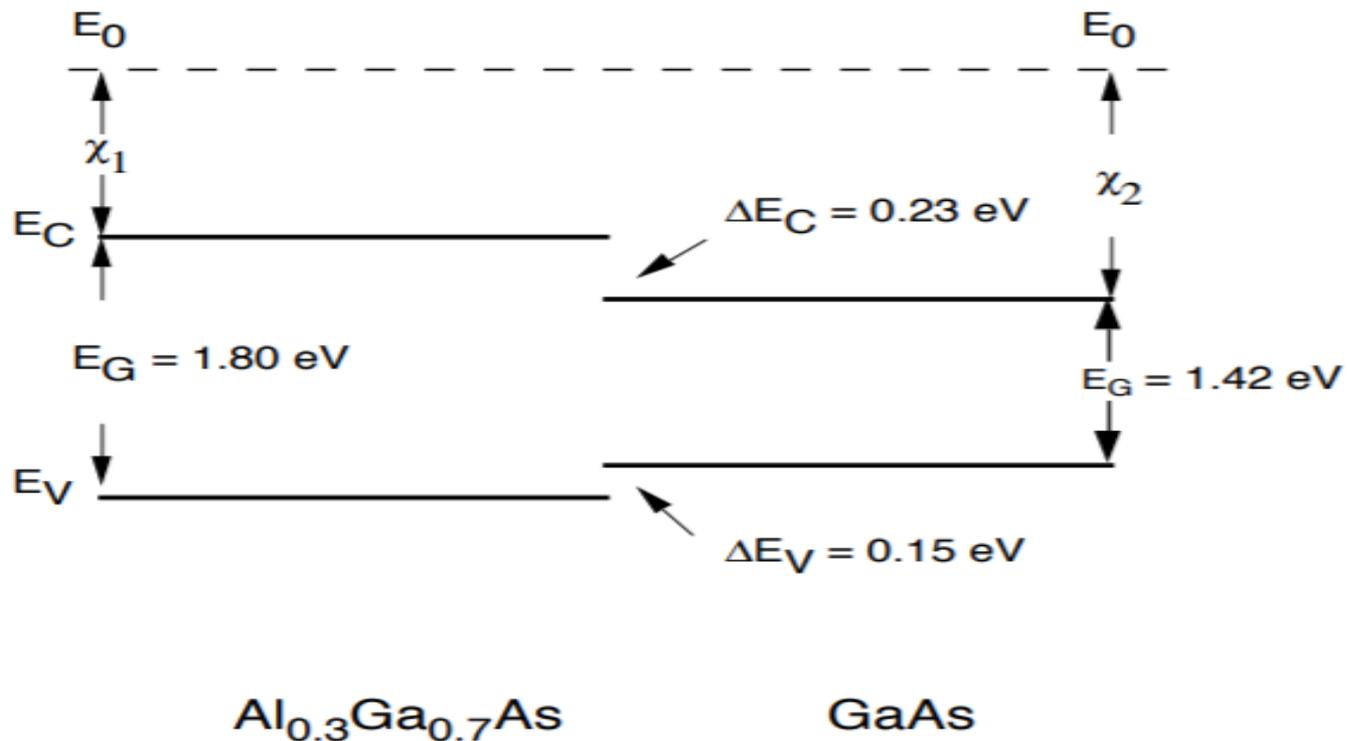
- In a typical SEM, an electron beam is thermionically emitted from an electron gun fitted with a tungsten filament cathode.
- The electron beam, which typically has an energy ranging from 0.2 keV to 40 keV, is focused by one or two condenser lenses to a spot about 0.4 nm to 5 nm in diameter. The beam passes through pairs of scanning coils or pairs of deflector plates in the electron column, typically in the final lens, which deflect the beam in the x and y axes so that it scans in a raster fashion over a rectangular area of the sample surface.
- The primary electron beam interacts with the sample, the electrons lose energy by repeated random scattering and absorption within a teardrop-shaped volume of the specimen known as the interaction volume, which extends from less than 100 nm to approximately 5 μm into the surface.

- The size of the interaction volume depends on the electron's landing energy, the atomic number of the specimen and the specimen's density.
- The energy exchange between the electron beam and the sample results in the reflection of high-energy electrons by elastic scattering, emission of secondary electrons by inelastic scattering and the emission of electromagnetic radiation, each of which can be detected by specialized detectors.
- Magnification in an SEM can be controlled over a range of about 6 orders of magnitude from about 10 to 500,000 times.

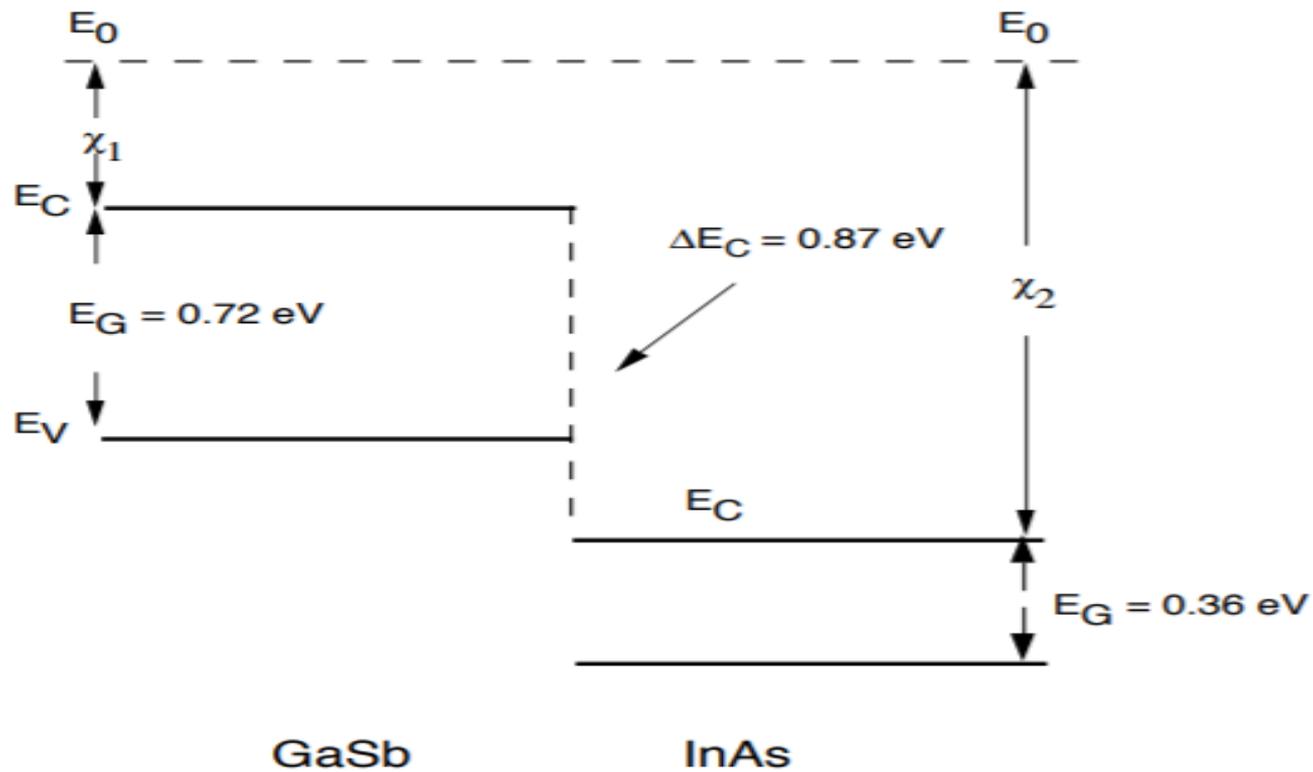
SEM Images



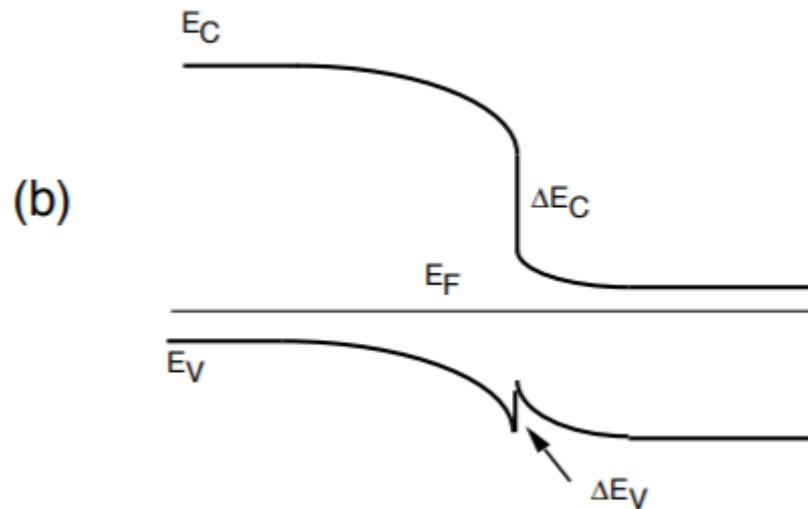
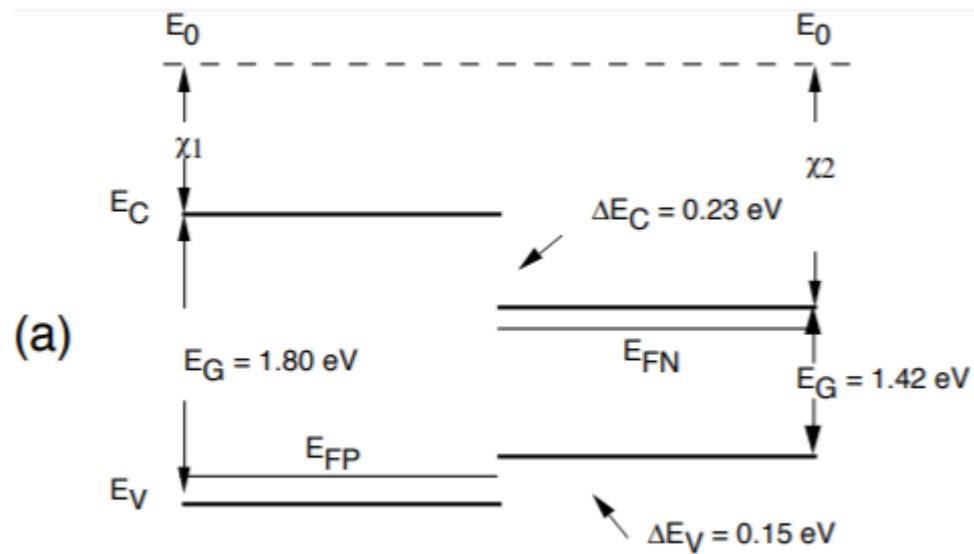
Heterojunctions and associated band diagram



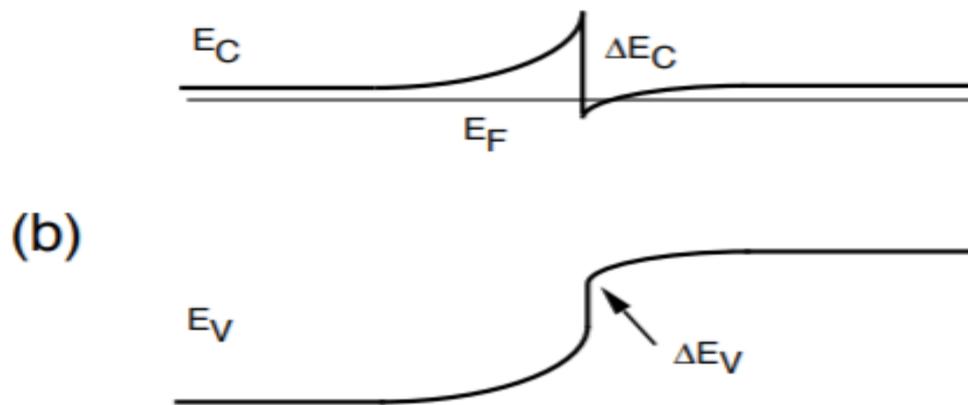
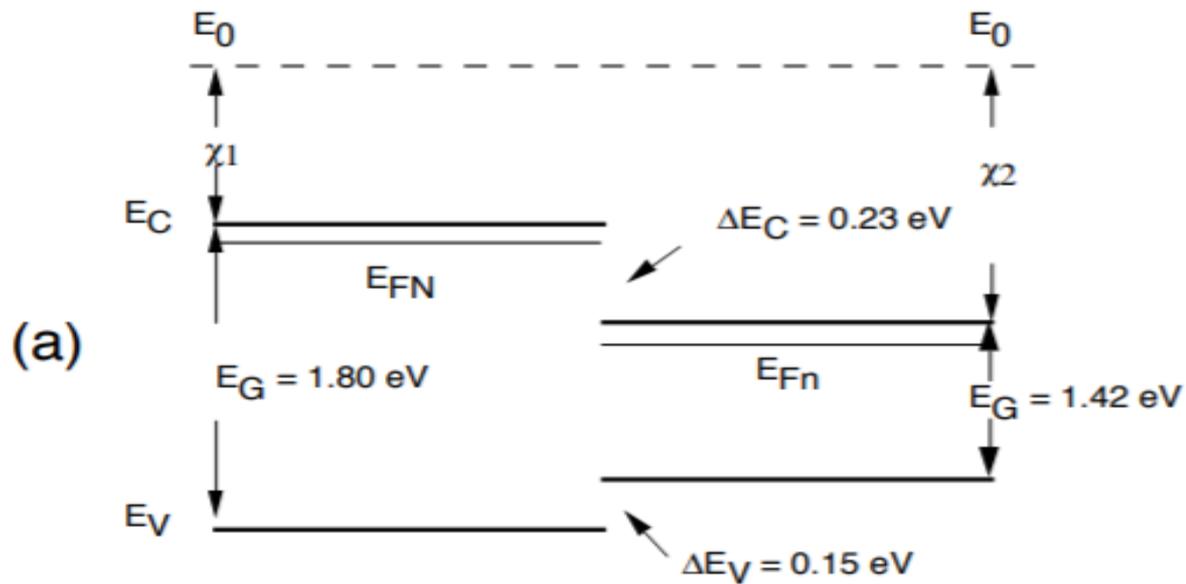
Type I Heterojunctions



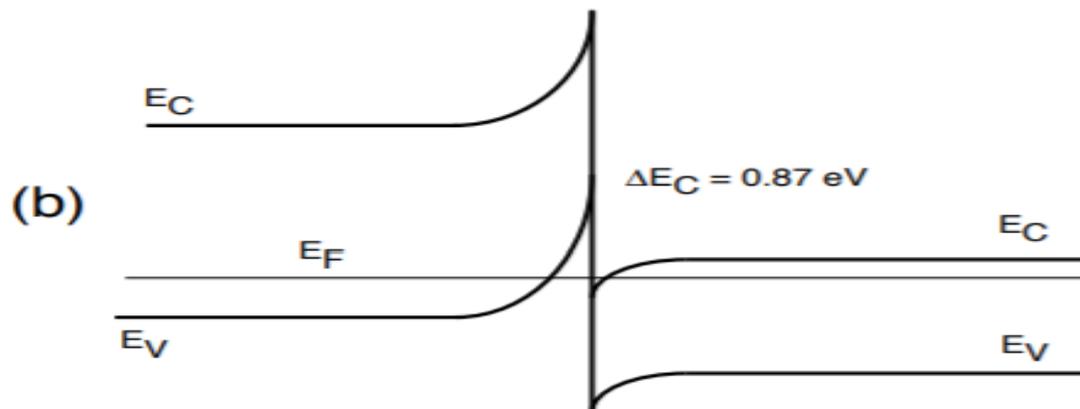
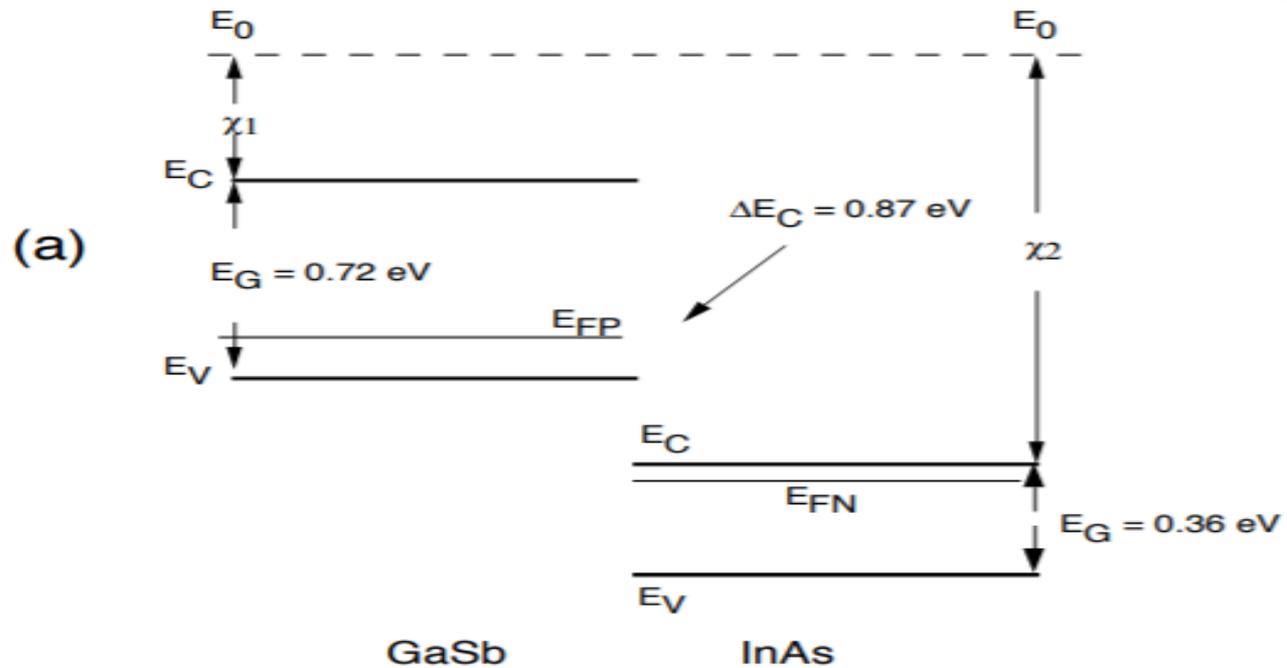
Type II Heterojunctions



$\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$: GaAs/ P-n heterojunction. (a) before contact. (b) after contact.



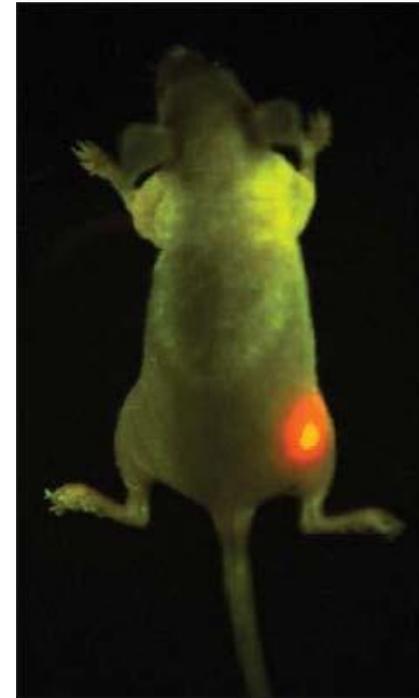
$\text{Al}_{0.3}\text{Ga}_{0.7}\text{As} : \text{GaAs} / \text{N-n}$ heterojunction. (a) before contact. (b) after contact.



GaSb : InAs/ P-n heterojunction. (a) before contact. (b) after contact.

Future Research

- Probe fundamental physics
- Quantum computing schemes
- Biological applications
- Improved Treatments for Cancer
- Optical and optoelectronic devices, quantum computing, and information storage.
- Semiconductors with quantum dots as a material for cascade lasers.
- Semiconductors with quantum dots as a material for IR photodetectors
- Injection lasers with quantum dots
- Color coded dots for fast DNA testing
- 3-D imaging inside living organisms



Bull's-eye. Red quantum dots injected into a live mouse mark the location of a tumor.

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